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1958

Space, Science, and Education
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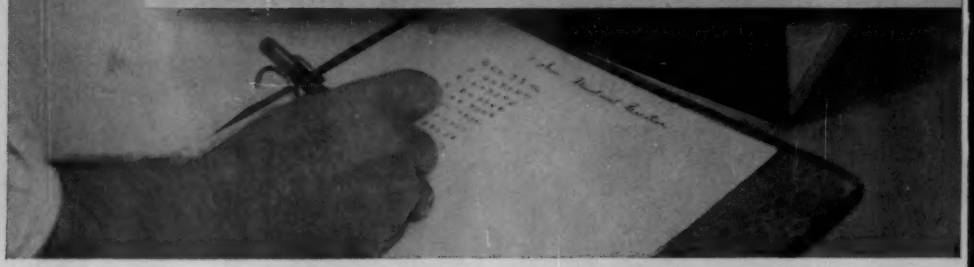

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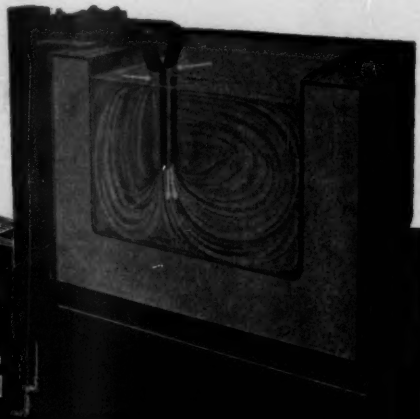

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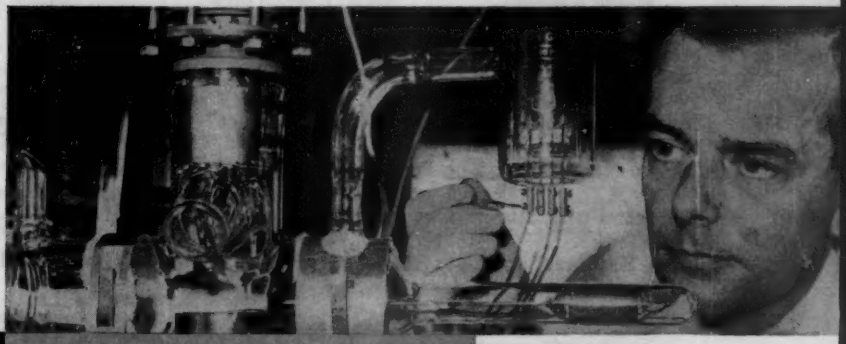

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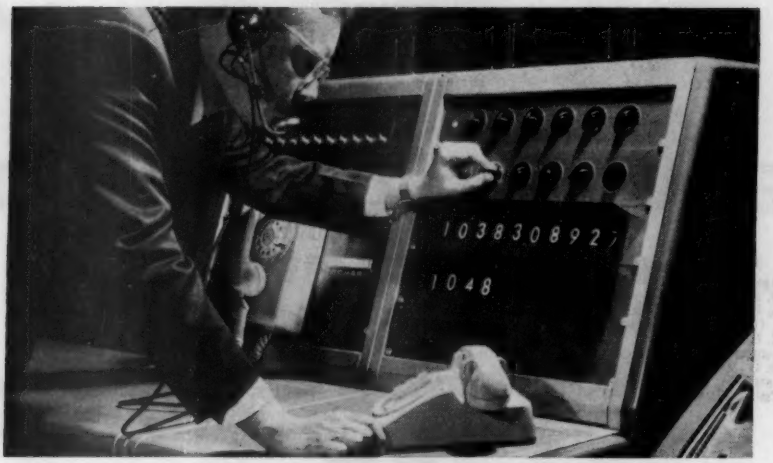

At monitoring console, designer H. D. Irvin watches performance of "Sibyl" during test of user-reaction to experimental telephones. A computer-like machine, Sibyl simulates the functions of future communications devices and records interplay between phones and users. Sibyl is named after the women oracles of ancient Greece.

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Vol. 266 DECEMBER, 1958 No. 6

SPACE, SCIENCE, AND EDUCATION *

BY

DONALD W. DOUGLAS [1]

President LePage, distinguished guests, members of The Franklin Institute, ladies and gentlemen:

When informed that I was to be a candidate for the Franklin Medal this year, I was greatly pleased but somewhat taken aback.

Being thoroughly mindful of the exacting standards which guide the Committee on Science and the Arts in selection of candidates for this high honor, I found it difficult to justify my presence in the company of such eminent former medalists as Edison, Orville Wright, Marconi, and Albert Einstein. That the Committee has seen fit to place an aircraft engineer turned aircraft and missile manufacturer in such distinguished ranks is a tribute for which I am humbly grateful.

As would have to be the case, those who have been chosen to receive previous Franklin Medals were, to a man, pioneers.

The pioneering aspects of *their* achievements are noteworthy and, it seems to me, fully in keeping with the long history and high traditions of The Franklin Institute.

John Quincy Adams was elected President the year after the Institute was founded. The Monroe Doctrine was only a year old and a goodly portion of the nation's peoples were pressing westward, exploring new territory, clearing the forests, turning the sod to prepare for the vast farm lands.

But in the early years of the Nineteenth Century, there were also

* Delivered at the 1958 Medal Day Meeting of The Franklin Institute, October 15, 1958, in acceptance of the Franklin Medal.

[1] Chairman of the Board, Douglas Aircraft Company, Inc., Santa Monica, Calif.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

many who were not primarily preoccupied with the conquest of the new nation's geographic frontiers. To such a group, Philadelphia was the place and 1824 the chosen time to give initial impetus to the examination of other values, to encourage pioneering in the fields of science and the mechanic arts.

The firm but gentle push which Samuel Vaughan Merrick, Dr. William Keating, and James Ronaldson provided 134 years ago was to generate a momentum which has increased steadily in power, a momentum which I think we all may devoutly hope, will continue to increase through many years to come.

Without a sustained intensification of this impulse to learning, to pioneering in science, engineering, and the mechanic arts, one may fear for the future of our hard-won national institutions.

I say this in the conviction that we have entered dramatically and almost explosively an era of great dependence on the scientist and the engineer.

Western civilization has known the Estate of the Clergy, that of the Nobles, the Third estate of the Commons and the Fourth estate of the Press. Each grew to power and made lasting impacts on the institutions and way of life of the time. None seemed to hold more promise than the present and coming ascendancy of the engineer.

The pace of the technological revolution speeded by the genius of such men as Edison, Urey, Whittle—and many others—is sweeping forward and upward with a fierce vitality. The scientists and engineers are today clearing the frontiers of space. The fields of past learning are being weeded of errors and misconceptions. No longer does it appear that man's fate is to spend the balance of his time in the confining atmospheric bonds of the present, "chained to this rock and lost in the stars."

But if our habitat inexorably is to be extended, if a limitless new dimension is to be added to our way of living, the probe of that destiny will surely depend to a large extent upon the findings of pure research, coupled with the ability of many specialized engineers to weld the yield of this research into the complex vehicles of space penetration.

I say "many specialized engineers" advisedly, because the past ten years have seen a growth and breadth of engineering development in our industry far beyond any comparable advance in an equivalent decade.

Today, missiles engineering has a requirement for well trained, experienced engineers and scientists in no less than eleven major categories, including, in addition to aeronautical experts, architects, chemists, metallurgists, nuclear physicists, and mathematicians. Within the major categories, we must have access to the talents of more than thirty different classes of specialists, each an expert in his own field.

Basically, this requirement for more specialized talents may be attributed to the advent of the missile age.

But that, I think, is an over-simplification, and deserves a bit of analysis.

Today's most pressing aim in the look to outer space, is toward the admittedly difficult—perhaps unattainable—goal of full performance reliability. We want to hit—or orbit—the moon, but we don't want to do it as the result of one lucky shot.

Hence, missile design and construction have brought the need for much more advanced research by the aircraft companies themselves, work that in the past has been shared with the National Advisory Committee for Aeronautics, that agency, however, carrying the major share of the load.

It has become increasingly necessary to keep close watch on research developments country-wide, because with the "systems" concept, the lack of one perfected component can effectively block the completion of a system design.

If a weapon system is under consideration, this lack makes it impossible to offer the using military service a fully developed operational proposal, which is to say no proposal can be made.

Hence, the increasing need for an accelerated "do-it-yourself" research program. Some advances come so fast that we simply can't wait for the outside development of the missing component.

This need for a greatly increased volume of independent research will require the talents and the best efforts of a highly educated and truly dedicated team of men and women, scholars who must sustain enthusiasm for their explorations in the knowledge that of all basic research, 95 per cent may be negative, with only 5 per cent productive.

Which brings us to what I regard as a most serious matter, a realistic appraisal of our educational system.

It finally has been recognized that the state of the nation cannot be evaluated without including education as one of the main yardsticks.

We owe this new situation to the Russians with their supersonic fighters, their big jet transports, their nuclear explosions and their satellites. Across the skies over every farm and city in our land, they have written the ancient truism that "Knowledge is Power."

It is for us vital that the nature of this change is generally understood and suitable action promptly taken.

To this end various groups in recent months have been analyzing our educational system and all of the practices, facilities and people involved in it. They have criticized methods and revised techniques. They have made motivation surveys among the children, argued about federal subsidy, devised new aptitude tests, and sought opinions and statistics in all walks of life, even to the point of asking advice from people who, like myself, employ engineers and scientists.

Being an engineer, I must respond by first pointing out that there is utterly no basis for assuming that our educational system has failed us

in any way. I can make this assertion safely because we never have had any generally accepted criteria for the end products. There is a saying attributed to a great scientist, Lord Kelvin, to the effect that "If you can't measure it, you don't know what you're talking about."

Without such a yardstick, our teaching policies could remain matters of opinion indefinitely, and probably never would be mentioned in connection with the state of the nation. The system could drift through successive experiments in the name of progress, without the means of measuring that progress. The result, being leavened by the vagaries of free enterprise competition, would lead to no conclusion unless one were willing to measure the product in dollars.

Now we have had thrust upon us, for the first time, a yardstick of our educational achievements. We are confronted with a record of Russian technical progress that sets some hard new goals for us.

Looking more closely at this record, we see that the real problem and threat is not their present status, *but in their momentum.*

The threat of world domination in Hitler's "Mein Kampf" was no clearer when it first appeared than the threat that is implicit in the possession of a completely modern technology by the ever-increasing masses of the Russian people who, until 25 years ago, were living in the atmosphere of the Dark Ages.

It hasn't been much over 25 years since Russia was offering triple salaries and careers of ease to engineers and skilled craftsmen who would migrate from America and show them how to build dams, fabricate plumbing systems and operate the simplest kinds of factories.

Yet in that quarter century—in a single generation—Russia has managed to absorb the whole technology of western civilization and move on ahead in significant areas. They have their share of geniuses.

A tremendous momentum—regimented scholastic effort—was created for which the West has no ready counterpart. Nothing can take us back suddenly to the invention of the steam engine and then permit us to actually experience, in rapid sequence, all of the history of modern science. Nothing can give us the advantage of exploring thousands of theories with the assurance that each has already been confirmed empirically—and with the actual hardware proof before us.

If this were possible, the effect would be an inspiration of such breadth and power as to draw invention from the mind of the dullest student.

We would proceed step by step through the conception and construction of the first blast furnace as if it were getting its first coke tomorrow. We would participate in designing the first harvester, the first standardized machine parts, the first volume production assembly line. We would watch with the Curies for the first glow of radium, and join with Newton in the perfection of calculus, with Kepler and Archimedes standing by as contemporaries.

Somehow, we must reveal more imaginatively the glamor that will draw young minds into science at an earlier age. By some new handling of the exciting history of science and by making certain that ability and ambition are encouraged, we can match or improve the thrills and triumphs that the Russians have found in their "catching up" process. For now, in terms of momentum and national dedication, it is we who must "catch up."

I don't believe the answer is in scholarships alone although these occasionally save a promising talent that would otherwise go without further development.

Part of the answer is public backing for ample school plants in all corners of the country, financed under conditions that make federal control of education an entirely separate controversy at the policy level, and not a qualification for brick and mortar. Let's make sure of the food before we bicker over the flavoring. This isn't centralization; it is simply unity of purpose.

Most of all, we need higher salaries, exemplary salaries, for the people who have the special talent and training to teach. We cannot depend only upon those whose call is so strong that they would starve to fulfill it. Here as elsewhere we will, in general, get only what we pay for.

May I suggest that to meet the Russian challenge, we might go further than we have in establishing incentive for the real inventors and innovators.

The Russians have the built-in advantages which accrue from a low-standard economy in which the special social and economic privileges granted their professional people give comforts which would not loom large in the eyes of their Western world counterparts.

In addition, however, major accomplishments are rewarded in very tangible fashion and I would like to advocate a similar practice. Sir George Edwards, managing director of Vickers-Armstrong and president of the Royal Aeronautical Society points out that Tupolev and other Russian aircraft designers have received from the state large tax-free grants. The British government saw fit to reward Whittle with a substantial honorarium and a knighthood.

To the best of my knowledge, the only comparable award in recent years made in this country was the $100,000 which the Congress voted William F. Friedman, the modest civilian cryptographer who broke the Japanese "purple" code during the last war.

I would propose that the Federal Government consider the establishment of an awards system, possibly patterned after the Nobel prizes, but preferably recognizing more categories germane to the space age. An equivalent of the $9,000,000 with which the Nobel prizes were endowed is surely not beyond the means of the government, which is to say, the American people.

I have a deep conviction that those people are wrong who think that

this heavy emphasis on the sciences is dangerous, and may produce ethical blindness, ignorance of human values, and brutish materialism.

Historically, learning in the physical sciences and learning in the humanities have seldom proceeded independently.

A revived thirst for knowledge, such as we have the opportunity now to foster as a national duty, will not be confined to technical matters.

It will merge into a revitalized respect for all learning, and finally for wisdom itself. It is this very formula—this certainty of eager migration from one field of knowledge to another—that mass psychologists tell us may eventually subvert the Russian technical community and turn them against their masters.

The Russians have learned the processes of scientific inquiry. They have the trained technicians—and the occasional genius—to continue this inquiry at a fast rate. They, too, will have their scientific break-throughs in such exotic fields as anti-gravity, weather control, controlled hydrogen fusion, and space flight; and mingled with these pursuits that have recognizable military significance they are not neglecting pure research, which by definition rejects arbitrary direction.

Thus, the challenge extends across all of the sciences, for this is not a weapons race in the traditional sense. We are engaged in a contest of total technologies or societies, testing whether in the long run learning itself can best be furthered by free men or by police state slaves.

A recognized superiority in modern science, which could snowball out of the present situation, would put Russia in a position to dictate a radically different way of life to a "world of satellites," perhaps without striking a blow.

This leaves us no alternative to an all-out educational mobilization. Our situation is not fundamentally different from that of the Romans when they discovered that the Visigoths had armed themselves with swords, shields and spears, and were about to sack the Empire.

Weapons are often different in form but never in purpose. Whole civilizations have died before when they underestimated the elemental forces and momentum that can arise overnight in some hinterland.

I am not one to minimize the difficulties ahead. A backward look over more than 40 years in the aircraft industry enforces the conviction that while the future may hold many "improbables," there will be very few "impossibles."

The scientist of today must know—and be able to practice—technologies literally unknown a very few years ago. We speak a language new to mankind. Selected at random, consider: Space age; nose cone; countdown; launching complex; thermonuclear; fall-out; mushroom cloud; weapon system; lunar probe. Here we have the terminology of today, words and expressions which did not even exist twenty years ago in their present connotations.

Whether the bulk of our people will be skeptical or optimistic as we continue the exploration of space, I cannot say. There is a tendency to shrug off the efforts of our men and women of science with a "So what?" I am confident that such an unrealistic attitude will in no degree discourage the men and women of science who are engaged in learning the cosmic alphabet, the syntax of space, the grammar of the spheres.

Since the day of Daedalus and Icarus, man has been drawn toward the stars. Historically, the failure of Icarus is perhaps better remembered than the success of his father. To me, they should be held in equal esteem, for I honor the courage of the experimenters over the failure of the experiment. May their courage never wane.

Metapolyphenyl Ethers Used in Nuclear Propulsion Systems.—General Electric chemists have developed base stocks for hydraulic fluids and lubricants for an aircraft nuclear propulsion system, which can withstand higher temperatures and radiation dosages than any fluid stocks in common use today, according to D. R. Shoults, the general manager of the Company's Aircraft Nuclear Propulsion Department (ANPD). The new fluids, for use in ANPD hydraulic systems and bearings, were developed in the company's General Engineering Laboratory, Schenectady, N. Y., through ANPD sponsorship, under Atomic Energy Commission and U. S. Air Force contract. Other companies carried out similar programs for general Air Force use.

Shoults said that these fluids, called metapolyphenyl ethers, could find applications also in supersonic equipment, such as rockets and space vehicles, wherever temperatures and radiation are critical. In addition, the materials hold promise as liquid insulation or dielectric liquids in commercial electrical apparatus.

The organic chemistry staff at the General Engineering Laboratory first selected a small number of possibilities that should be thermally resistant, based on inherently radiation-resistant structure and freedom from unstable groups of atoms. After months of synthetic and testing work, the metapolyphenyl ethers formed one of three series of materials for further development and finally proved to be the most stable thermally.

Paper Sandbag.—A paper sandbag that, in time of emergency could replace the standard jute sandbag with respect to quality and supply is under development at the U. S. Army Engineer Research and Development Laboratories, Fort Belvoir, Virginia. As part of this development program, 26,000 paper sandbags are now being tested at various locations.

The knitted paper sandbag under development has all of the physical qualities of its jute counterpart, both wet and dry, and also stacks and handles satisfactorily. The sandbag mesh does not lose any significant quantity of fill material, except when a very dry fine sand is used. The mesh, however, can be made as fine as desired. Excellent wet strength is obtained by treatment with a urea-formaldehyde resin (Uformite). Fungicidal protection is presently achieved by use of a form of solubilized Copper 8 Quinolinolate.

On the basis of restricted field tests, the knitted paper sandbag can be classified as a 60-day bag under the most severe weathering conditions, and can withstand normal weathering effects in excess of 6 months. The knitted fabric does not ravel when punctured; resists the shock effects of a close blast, at least as well as jute burlap, and has a service life in water comparable to that of the standard military jute sandbag.

The present stage of paper sandbag development has been reached through the cooperation of the Cellucord Corp., Gilman, Vermont and Commonwealth Coloring and Chemical Co., N. Y. C., who supplied the experimental knitted paper bags and copper 8 quinolinolate fungicide formulation, respectively.

It is expected that with further development work, the service life of these paper sandbags will be significantly increased.

A POYNTING THEOREM FOR MOVING BODIES AND THE RELATIVISTIC MECHANICS OF EXTENDED OBJECTS

BY

NORMAN H. MEYERS[1]

ABSTRACT

Maxwell's macroscopic field equations have long been regarded as the correct formulation of electrodynamic behavior even in the presence of moving materials. However, they do not lead to a Poynting theorem which can be given a reasonable physical interpretation with moving media present. The power converted to mechanical form, for example, is lost somewhere in the usual Poynting theorem. This fact among others has led recent workers in classical macroscopic electrodynamics to the conclusion that Maxwell's equations are not sufficiently general. However, the special theory of relativity seems to require that they hold in all inertial reference frames since they agree with experiment in a proper or rest frame. This idea is an abuse of the special theory of relativity. It is pointed out in this paper that a law of physics can only be regarded as invariant in form and meaning for all inertial observers if it agrees with experiment in a non-proper or moving frame of reference. The physical laws take on degenerate forms in a proper frame since there is something unique about a proper frame which makes it different relativistically from all other frames. Thus, Maxwell's equations must represent a degenerate form of more basic relationships which are valid in all inertial frames.

Lan J. Chu has postulated a modified set of electrodynamic relationships which reduce to Maxwell's equations in a proper or rest coordinate system. It is shown that Chu's self-consistent re-formulation of macroscopic electrodynamics leads to a Poynting-type theorem which is easy to interpret physically even in the presence of moving media.

The thorough interpretation of this Poynting theorem for moving bodies requires a study of the meaning of force and power from a macroscopic point of view and the relativistic laws of transformation which they obey. An approach to the relativistic mechanics of extended objects is postulated. It is assumed that there exists for every object at a particular instant some inertial coordinate system in which the net momentum of the object is zero. Various parts of the object move relative to this rest coordinate system but the net momentum of all the parts vanishes. The total energy of the object in its rest frame is the rest energy. The rest energy or rest mass of the object may vary with time. The total energy and net linear momentum of the object in some moving inertial coordinate system are related to the rest frame quantities in exactly the same way as for a point mass. The variation with time of the body's rest mass does not destroy the covariance of the formulation. Furthermore, the approach developed is consistent with macroscopic electrodynamics. As an electromagnetic field heats or polarizes a body, its rest mass increases. Relativistic particle mechanics does not allow such an effect since the rest mass is regarded as an invariant property of the particle.

A. INTRODUCTION

Electromagnetic field theory in vacuum is a satisfying, unambiguous, self-consistent formulation which is in agreement with all experiments

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performed to date. The special theory of relativity has been successfully used to extend our understanding of vacuum electrodynamics. However, in the presence of material media the situation is not so clear. There is still some controversy as to whether **H** or **B** should be regarded as the fundamental vector associated with a magnetic field in the presence of material (1).[2] If the material is in motion relative to an observer the situation is even worse. None of the four vectors **E**, **H**, **D**, or **B** has a completely clear meaning.

In the presence of motionless material the macroscopic or large scale field equations of Maxwell are usually written as

$$\text{curl } \mathbf{E} = -\partial\mathbf{B}/\partial t \tag{1}$$

$$\text{curl } \mathbf{H} = \mathbf{J} + \partial\mathbf{D}/\partial t \tag{2}$$

$$\text{div } \mathbf{D} = \rho \tag{3}$$

$$\text{div } \mathbf{B} = 0. \tag{4}$$

The vectors **E** and **B** are regarded as field vectors defined by the Lorentz force.

$$\mathbf{F} = q(\mathbf{E} + \mathbf{u} \times \mathbf{B}). \tag{5}$$

Experimental verification of the Lorentz force has been carried out only in vacuum.[3] The vectors **H** and **D** are generally regarded as source quantities since, in a static case with appropriately chosen material geometries, **H** depends only on **J** and **D** depends only on ρ.

From the Maxwell equations a Poynting theorem can be derived. The result is

$$-\text{div } (\mathbf{E} \times \mathbf{H}) = \mathbf{E}\cdot\mathbf{J} + \mathbf{E}\cdot\partial\mathbf{D}/\partial t + \mathbf{H}\cdot\partial\mathbf{B}/\partial t. \tag{6}$$

The physical interpretation of the various terms in the Poynting theorem is based on certain well-known assumptions (3) which need not be restated here. The interpretations are as follows.

1. The quantity $-\nabla\cdot(\mathbf{E} \times \mathbf{H})$ is the net density with which electromagnetic power flows into a differential volume.

2. The quantity $\mathbf{E}\cdot\mathbf{J}$ is the power per unit volume expended by the electric field against the current **J**. This power may be dissipated as Joule heat, converted to kinetic energy of charged particles, or converted to chemical form as in the charging of a battery.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

[3] The Lorentz force has been measured on cosmic rays in a magnetic material by Rasetti (2). However, this work cannot be regarded as shedding much light on a macroscopic field theory because the particles involved were of such high velocity as to be able to penetrate the near field of the microscopic current loops which cause magnetic effects. Thus microscopic rather than large-scale macroscopic effects were involved in Rasetti's experiments.

3. The quantity $\mathbf{E}\cdot\partial\mathbf{D}/\partial t$ is the power per unit volume expended to build up an electric field in the presence of dielectric material. The validity of this term as a power density can of course not be proved.

4. The quantity $\mathbf{H}\cdot\partial\mathbf{B}/\partial t$ is analogously the power per unit volume expended to establish or increase a magnetic field in the presence of magnetic material.

Next consider the case of a moving material. The fundamental postulate of the special theory of relativity requires that all the laws of physics, if correctly formulated, be invariant in form and meaning for all inertial observers. Therefore, it seems that Maxwell's equations as well as the definitions of the vectors **E**, **H**, **D**, and **B** should remain unaltered by a change of the inertial observer's reference frame. The Poynting theorem in the form of Eq. 6 should remain unchanged. Its interpretation also ought to be the same in the presence of moving materials if the usual interpretation is sufficiently general. Let us examine this question further.

Power is converted to mechanical form when a polarizable material moves through an electromagnetic field because the field exerts a force on the body. Certainly power thus converted should appear as a part of the electromagnetic power balance expressed by Poynting's theorem. Just where this power converted to mechanical form is hidden in the terms of Poynting's theorem is not at once apparent. The Lorentz transformation of the field vectors (3, pp. 78–81) can be used profitably in the search. Consider the term $\mathbf{E}\cdot\mathbf{J}$, for example, and suppose that a conducting material moves past the inertial observer, who measures **E** and **J**, with a uniform velocity **u**. The Lorentz transformation enables one to relate the measurements of a non-moving observer with those of an observer riding along with the body.

$$\mathbf{E}\cdot\mathbf{J} = \gamma_u\mathbf{E}_0\cdot\mathbf{J}_0 + \gamma_u\mathbf{u}\cdot(\rho_0\mathbf{E}_0 + \mathbf{J}_0 \times \mathbf{B}_0). \tag{7}$$

The subscripts zero denote quantities measured by an observer instantaneously at rest with respect to the material. Such an observer is called a proper observer and his inertial space-time coordinate system is called the proper frame of reference. The apparent laboratory volume element dV of a moving body appears contracted in the direction of its motion by a factor of $1/\gamma_u$ so that

$$dV = dV_0/\gamma_u, \quad \text{where} \quad \gamma_u = (1 - u^2/c^2)^{1/2}. \tag{8}$$

Applying this to identity (7) there results

$$\int_V \mathbf{E}\cdot\mathbf{J}dV = \int_{V_0} \mathbf{E}_0\cdot\mathbf{J}_0 dV_0 + \mathbf{u}\cdot\int_{V_0}(\rho_0\mathbf{E}_0 + \mathbf{J}_0 \times \mathbf{B}_0)dV_0. \tag{9}$$

If the body is a linear conductor so that $\mathbf{J}_0 = \sigma\mathbf{E}_0$ the first term on the right side of Eq. 9 is the power converted to Joule heat as reported by a

proper observer. The second term on the right side of Eq. 9 is the scalar product of the proper frame force acting on the charges and currents associated with the body and the velocity of the body relative to the non-moving or laboratory observer. It will be proved later that this second term may under certain conditions be correctly regarded as the power converted to mechanical form as seen by the laboratory observer. Equation 9 is a power balance which requires that the power expended by the electric field against the current **J** in the presence of a moving conductor either be absorbed by the conductor or be converted to mechanical form. This is a satisfying result and leads one to hope that the same procedure can be applied to the other terms of Poynting's theorem.

Unfortunately no amount of juggling of terms or Lorentz transforming back and forth between frames leads to a reasonable interpretation of the terms $\mathbf{E}\cdot\partial\mathbf{D}/\partial t$ and $\mathbf{H}\cdot\partial\mathbf{B}/\partial t$ in Poynting's theorem. One should be able to separate these terms into power stored in the field and material and power converted to mechanical form as a result of electromagnetic forces on moving dielectric and magnetic material. In other words it should be possible to derive a Poynting theorem, valid in the presence of moving materials, which clearly separates the net electromagnetic power entering a volume into all the physically reasonable component parts. Physical reasoning requires the following division of the net electromagnetic input power.

Net electromagnetic power input	=	Power stored in electromagnetic field	+	Electromagnetic power stored in material

	+	Electromagnetic power dissipated as heat	+	Electromagnetic power converted to mechanical form

The recognition of the fact that the correct Poynting theorem in the presence of moving material must separate in this fashion is one of the important ideas of this paper.

B. THE SOURCE OF PAST ERROR

The difficulties encountered in attempting to separate the terms of the usual Poynting theorem into components with clear physical meaning lead one to wonder about the validity of Maxwell's equations in the presence of moving material. There is another reason to be found in the literature for doubting their general validity. The covariant formulation of the laws of conservation of electromagnetic energy and momentum leads to an unsymmetrical energy-momentum tensor (1, pp.

349, 350). This is also physically unsatisfying but attempts to resolve the difficulty were abandoned until recently.

Certainly the Maxwell equations are correct in a proper reference frame. There is a vast wealth of experimental evidence to support them. In addition there is little reason to doubt the postulate of special relativity which requires the invariance of all physical laws if correctly formulated. How can it be then that the physical laws expressed by Maxwell's equations seem to be in error in a non-proper reference frame? The explanation is simple. A law of physics can only be regarded as correctly formulated if it agrees with experiment in a non-proper frame. All inertial coordinate systems are equivalent in principle according to special relativity except the proper coordinate system. There is something unique about an inertial frame of reference which is proper with respect to a body at a particular instant. The physical laws may assume degenerate forms in a proper coordinate system. One can not recover the correct general law by requiring the invariance in form and in meaning for all inertial observers of the degenerate, proper-frame law. The usual formulation of Maxwell's equations in the presence of materials must be a degenerate case of a more general set of equations which are valid in all non-proper frames.

To expand more fully on the uniqueness of a proper coordinate system, consider a point mass in motion relative to two inertial observers. Let the mass move with velocity $\mathbf{u}$ relative to a laboratory or fixed observer and with velocity $\mathbf{u}'$ relative to an observer in a primed inertial reference frame. The uniform velocity of the primed frame

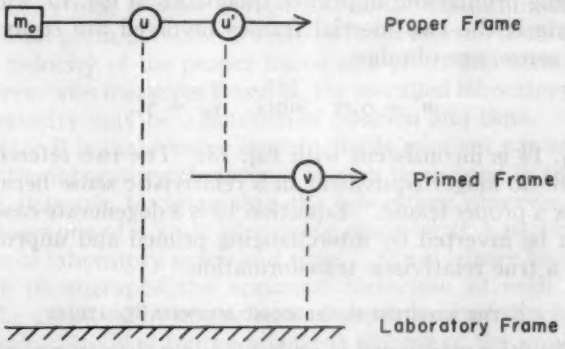

FIG. 1. Point mass in motion relative to two inertial observers.

with respect to the unprimed frame is $\mathbf{v}$ as shown in Fig. 1. The rest mass of the point object is m_0. Let c denote the velocity of light in free space. Let

$$\gamma_u = (1 - u^2/c^2)^{-1/2} \qquad \gamma_{u'} = (1 - (u')^2/c^2)^{-1/2} \qquad \gamma_v = (1 - v^2/c^2)^{-1/2},$$

The apparent mass of the body as seen by the laboratory observer is $m = \gamma_u m_0$ while the apparent mass reported by the primed observer is $m' = \gamma_{u'} m_0$. Thus

$$m_0 = m/\gamma_u = m'/\gamma_{u'}. \tag{10}$$

Equation 10 shows that the apparent laboratory mass can be expressed in terms of the apparent primed mass as

$$m = \gamma_u m'/\gamma_{u'}. \tag{11}$$

Since the laboratory and primed frames are entirely equivalent in principle according to special relativity, one may invert or solve any true relativistic transformation by interchanging primed and unprimed quantities and changing the sign of **v**. Applying this idea to Eq. 11

$$m' = \gamma_{u'} m/\gamma_u. \tag{12}$$

Equation 12 expresses the apparent primed mass in terms of the apparent laboratory mass. It should be noted that Eq. 12 may be substituted back into either Eq. 10 or 11 without contradiction. The transformation obtained by interchanging primed and unprimed quantities is therefore the correct inverse of the original transformation.

Next suppose that the primed frame is instantaneously a proper one so that $\mathbf{u}' = 0$, $\mathbf{u} = \mathbf{v}$, $\gamma_{u'} = 1$, and $\gamma_u = \gamma_v$. Equation 11 becomes

$$m = \gamma_v m'. \tag{13}$$

Interchanging primed and unprimed quantities in Eq. 13, which should be legitimate if the two inertial frames involved are equivalent in a relativistic sense, one obtains

$$m' = \gamma_v m \quad \text{since} \quad \gamma_{v'} = \gamma_v. \tag{14}$$

Clearly Eq. 14 is inconsistent with Eq. 13. The two reference frames involved are no longer equivalent in a relativistic sense because one of them is now a proper frame. Equation 13 is a degenerate case of Eq. 11 and cannot be inverted by interchanging primed and unprimed quantities as in a true relativistic transformation.

C. THE MODIFIED MACROSCOPIC MAXWELL EQUATIONS

The difficulties mentioned in understanding the macroscopic electrodynamics of moving media led Lan J. Chu to postulate a modified set of electrodynamic equations (4). Chu's equations relate the field vectors **E** and **H** measured in any non-proper inertial frame of reference. They reduce to the usual Maxwell equations in a proper frame. The implication is that Chu's equations are the formulation of a law of physics and that Maxwell's equations are a degenerate, special, proper-

frame case. The field equations as modified by Chu are given as follows. Rationalized MKS units are assumed throughout this paper.

$$\text{curl } \mathbf{E} = -\partial \mu_0 \mathbf{H}/\partial t - \mathbf{J}^* \tag{15}$$

where $\mathbf{J}^* = \partial \mu_0 \mathbf{M}/\partial t + \text{curl } (\mu_0 \mathbf{M} \times \mathbf{u})$.

$$\text{curl } \mathbf{H} = \mathbf{J} + \partial \epsilon_0 \mathbf{E}/\partial t + \mathbf{J}_p \tag{16}$$

where $\mathbf{J}_p = \partial \mathbf{P}/\partial t + \text{curl } (\mathbf{P} \times \mathbf{u})$.

$$\text{div } \epsilon_0 \mathbf{E} = \rho + \rho_p \tag{17}$$

where $\rho_p = -\text{div } \mathbf{P}$.

$$\text{div } \mu_0 \mathbf{H} = \rho^* \tag{18}$$

where $\rho^* = -\text{div } \mu_0 \mathbf{M}$.

These are macroscopic equations which apply to smeared or averaged quantities. The field vectors **E** and **H** are regarded as defined by the macroscopic force law

$$\mathbf{F} = q(\mathbf{E} + \mathbf{u} \times \mu_0 \mathbf{H}) \tag{19}$$

even in the presence of moving material. Such a definition of **E** and **H** is consistent with the Lorentz force of Eq. 5 only in vacuum, but bear in mind that macroscopically speaking the Lorentz force has to date been checked experimentally only in vacuum where **B** equals $\mu_0 \mathbf{H}$.

The velocity **u** at a point is regarded as the instantaneous velocity of a differential grain of material relative to an inertial observer. That is, **u** is the velocity of the proper frame at a point and instant relative to the observer who measures **E** and **H**, the so-called laboratory observer. Thus the velocity may be a function of position and time.

The vector **P** is the average electric dipole moment per unit volume as seen by the laboratory observer. It may be defined in the presence of moving dielectric by imagining the laboratory observer to take a set of photographs of a grain of moving material at a particular point and instant of laboratory space and time. At his leisure he may measure on the photographs the apparent distortion of each individual dipole in the grain. The vector sum of the individual dipole moments thus determined divided by the apparent volume of the grain given by the photographs yields **P** of the grain. The size of the grain under discussion is such that it contains many dipoles but is small enough to be of differential dimensions in a macroscopic theory.

The magnetic dipole moment **M** can be regarded as defined in a similar manner. There are some subtleties involved in working with the concept of magnetic dipoles which Chu discusses in his paper (4).

If Chu's equations are the correct formulation of the classical macro-

scopic laws of electrodynamics in any non-proper inertial frame, the special theory of relativity requires that they hold in form and meaning for all other inertial observers. In other words, if Eq. 15 holds in the lab frame of Fig. 1, it must also apply in the primed frame of Fig. 1.

$$\text{curl}' \mathbf{E}' = -\partial \mu_0 \mathbf{H}'/\partial t' - \mathbf{J}^{*\prime} \tag{20}$$

where $\mathbf{J}^{*\prime} = \partial \mu_0 \mathbf{M}'/\partial t' + \text{curl}' (\mu_0 \mathbf{M}' \times \mathbf{u}')$.

The same is true for the other three equations. This invariance in form and meaning combined with the Lorentz transformation of space-time coordinates is enough information to uniquely relate the primed field and material quantities to the laboratory quantities. The results are

$$\mathbf{E}_\perp' = \gamma_v(\mathbf{E}_\perp + \mathbf{v} \times \mu_0 \mathbf{H}) \qquad \mathbf{E}_\parallel' = \mathbf{E}_\parallel \tag{21}$$

$$\mathbf{H}_\perp' = \gamma_v(\mathbf{H}_\perp - \mathbf{v} \times \epsilon_0 \mathbf{E}) \qquad \mathbf{H}_\parallel' = \mathbf{H}_\parallel \tag{22}$$

$$\mathbf{J}_\parallel' = \gamma_v(\mathbf{J}_\parallel - \rho \times \mathbf{v}) \qquad \mathbf{J}_\perp' = \mathbf{J}_\perp \tag{23}$$

$$\mathbf{P}_\perp' = \mathbf{P}_\perp/\gamma_v \qquad \mathbf{P}_\parallel' = \mathbf{P}_\parallel \tag{24}$$

$$\mathbf{M}_\perp' = \mathbf{M}_\perp/\gamma_v \qquad \mathbf{M}_\parallel' = \mathbf{M}_\parallel \tag{25}$$

$$\rho' = \gamma_v(\rho - \mathbf{v}\cdot\mathbf{J}/c^2). \tag{26}$$

The symbol $\perp$ denotes components perpendicular to $\mathbf{v}$ while $\parallel$ denotes components in the direction of $\mathbf{v}$. The transformations above are quite different from the ones which result by requiring the invariance of the usual Maxwell equations (3, pp. 78–81).

The postulates of Chu combined with his modified version of the equations of electrodynamics lead to a symmetrical energy momentum tensor. Furthermore, as will be shown, they lead to a Poynting theorem which is easy to interpret physically in the presence of moving material. Some points in Chu's work may still require experimental verification but the outcome of such experiments has no real bearing on the present discussion. The important point here is that whichever the correct equations of electrodynamics turn out to be they must yield a Poynting theorem which separates into all the physically recognizable components of power. Chu's equations are used to illustrate how this works out with a self-consistent formulation of electrodynamics.

D. THE MODIFIED POYNTING THEOREM

Operation on the two curl expressions, Eqs. 15 and 16, in the usual manner gives a Poynting-type theorem

$$-\text{div}\,(\mathbf{E} \times \mathbf{H}) = \partial w/\partial t + \mathbf{E}\cdot\mathbf{J} + \mathbf{E}\cdot\mathbf{J}_p + \mathbf{H}\cdot\mathbf{J}^* \tag{27}$$

where $w = \epsilon_0 E^2/2 + \mu_0 H^2/2$.

It is not at once apparent where the terms representing power stored or dissipated in moving material and power converted to mechanical form are concealed in the new theorem. The technique used earlier in Eq. 7 can be applied here. Using Eqs. 21 through 26 to transform the unidentifiable terms in Eq. 27 into the proper frame effects the desired separation. The subscripts zero again denote proper frame quantities, except for μ_0 and ϵ_0 which are the free space permeability and permittivity.

$$-\operatorname{div}(\mathbf{E}\times\mathbf{H}) = \partial w/\partial t + \gamma_u {}^{*}p_0 + \gamma_u \mathbf{u}\cdot{}^{*}\mathbf{f}_0 \tag{28}$$

where

$${}^{*}p_0 = \mathbf{E}_0\cdot\mathbf{J}_0 + \mathbf{E}_0\cdot\partial\mathbf{P}_0/\partial t_0 + \mathbf{H}_0\cdot\partial\mu_0\mathbf{M}_0/\partial t_0 \tag{29}$$

and

$${}^{*}\mathbf{f}_0 = \mathbf{E}_0(\rho_0 + \rho_{p0}) + (\mathbf{J}_0 + \partial\mathbf{P}_0/\partial t_0)\times\mu_0\mathbf{H}_0 + \mathbf{H}_0\rho_0^{*} - \partial\mu_0\mathbf{M}_0/\partial t_0 \times \epsilon_0\mathbf{E}_0. \tag{30}$$

The force density ${}^{*}\mathbf{f}_0$ represents the force of electromagnetic origin per unit proper volume which acts upon the free charges, electric polarization charges, magnetic polarization charges, and their associated currents in the proper frame. It is one of the important postulates of the Chu formulation of electrodynamics. The term ${}^{*}p_0$ is the proper frame power density associated with conduction loss as well as with storage in electric and magnetic dipoles.

Integration of the point relationship (28) throughout the volume of a differential grain of material merely amounts to multiplication by the volume element dV. Since a volume element appears to shrink in the direction of its motion

$$dV = dV_0/\gamma_u. \tag{31}$$

Letting

$${}^{*}\mathbf{f}_0 dV_0 = {}^{*}\mathbf{F}_0 \quad \text{and} \quad {}^{*}p_0 dV_0 = {}^{*}P_0, \tag{32}$$

Eq. 28 becomes

$$-\operatorname{div}(\mathbf{E}\times\mathbf{H})dV = (\partial w/\partial t)dV + {}^{*}P_0 + \mathbf{u}\cdot{}^{*}\mathbf{F}_0. \tag{33}$$

If the modified Poynting theorem in Eq. 33 is to be regarded as an expression of the conservation of electromagnetic power, each term must be given a physical interpretation in the laboratory coordinate system. The conservation of energy or power is a concept which has meaning only in a single inertial frame of reference. Thus ${}^{*}P_0$ must be given a meaning for a laboratory observer even though it is expressed in terms of proper frame field and material quantities. The physical interpretations which can be given to the terms in (33) are as follows:

1. $-$ div $(\mathbf{E} \times \mathbf{H})dV$ is the net electromagnetic power entering the motionless laboratory volume element dV.

2. The quantity $(\partial w/\partial t)dV$ is the power stored in the electromagnetic field in the volume element dV. All the usual difficulties with the idea of a power density are still present here.

The remaining two terms ${}^{s}P_0$ and $\mathbf{u}\cdot{}^{s}\mathbf{F}_0$ can be given simple physical meanings for a laboratory observer under two special conditions. The first restriction is that the grain in question must move at constant velocity relative to the laboratory. Secondly, the grain must be mechanically rigid so that no energy can be stored in it as a result of its mechanical deformation. This is not to be construed as ruling out electromagnetic polarization. The grain may be polarized. If these two conditions are met,

3. ${}^{s}P_0$ is the total power soaked up by the moving grain as seen by a laboratory observer. It includes not only the apparent power absorbed by the moving grain in the form of heat and dipole energy but also a component representing an increase in the momentum of the grain. Even though constant velocity has been assumed the momentum can increase because the rest mass can increase as will be explained in detail later.

4. The term $\mathbf{u}\cdot{}^{s}\mathbf{F}_0$ is the power converted to mechanical form and extracted from the electromagnetic system by a mechanical force source, as seen by a laboratory observer.

The interpretations given to ${}^{s}P_0$ and $\mathbf{u}\cdot{}^{s}\mathbf{F}_0$ must next be justified. To accomplish this it is necessary to examine the nature of forces and power and the laws of transformation which they obey. It will be found that the terms ${}^{s}P_0$ and $\mathbf{u}\cdot{}^{s}\mathbf{F}_0$ can be interpreted without the restrictions of rigidity and constant velocity. However, the interpretations must then be less specific.

E. EXTENSION OF RELATIVISTIC PARTICLE MECHANICS TO COMPLETE BODIES

The laws of relativistic mechanics are thoroughly developed for point masses. In pre-relativistic mechanics the rigid body approximation was found very useful in dealing with extended objects. Unfortunately, the extended rigid body has no counterpart in relativistic mechanics (5). The internal rigid body constraints have never been successfully formulated in a Lorentz covariant manner.

In this section the ideas of relativistic particle mechanics will be logically extended to cope with a system of particles somehow bound together into a body. Some reasonable sounding assumptions and postulates will be made which do not destroy the covariance of the results. In other words the approach taken is consistent with the requirements of special relativity. Essentially, an extended body is treated as a particle having a variable rest mass. Only linear momenta and forces of translation are considered. The concept of a variable

rest mass enables one to reconcile macroscopic electrodynamics with mechanics for extended bodies. In the present state of the art, electrodynamics is consistent with relativistic mechanics only for dimensionless particles.

A brief review of the salient features of relativistic particle mechanics is in order at this point. It is customary to require that the law of conservation of momentum be maintained in its classical form. Thus the linear three dimensional momentum of a point mass m is given by

$$\mathbf{p} = m\mathbf{u}. \tag{34}$$

The momentum $\mathbf{p}$ is a three dimensional space vector as is the velocity of the particle $\mathbf{u}$. Neither $\mathbf{p}$ nor $\mathbf{u}$ is the spatial part of a four-vector if the mass is independent of velocity. Therefore, physical laws formulated using these quantities will not necessarily be Lorentz covariant. This simply means that the laws will not be the same in form and in meaning for all inertial observers, as is required by special relativity. However, the ordinary linear three-dimensional momentum of Eq. 34 becomes the spatial part of a four-vector if the mass is regarded as a particular function of the velocity of the particle relative to an inertial observer (1, pp. 267, 268). The function required is

$$m = \gamma_u m_0, \tag{35}$$

where m_0 is the rest mass of the particle. The fourth component of the four-vector momentum thus generated involves the energy of the particle. The contravariant four-momentum may be written as

$$\mathbf{p}^i = (\gamma_u m_0 \mathbf{u}, \gamma_u m_0 c). \tag{36}$$

The quantity c is the velocity of light in free space. It is an invariant. A four-vector is formed merely by building up a quantity whose components transform from one inertial frame to another like the coordinates of a point in four-dimensional space.

The fourth component of the four-vector in Eq. 36 is the total energy of the particle divided by c. One may deduce this as follows. In order to preserve Newton's law of force it is desirable to continue to define the net three-dimensional force acting on the particle as the rate of change of the three-dimensional momentum. Thus, by definition

$$\mathbf{F} = d(\gamma_u m_0 \mathbf{u})/dt. \tag{37}$$

The law of conservation of energy requires that the power expended by the net applied force acting on the particle be absorbed as a rate of increase of the particle's apparent energy. Therefore

$$P = \mathbf{u} \cdot \mathbf{F} = dE/dt, \tag{38}$$

where E is the total energy of the particle as seen by the same inertial observer who records the force $\mathbf{F}$. With the rest mass treated as a constant, one can readily demonstrate that

$$P = \mathbf{u}\cdot\mathbf{F} = d(\gamma_u m_0 c^2)/dt. \tag{39}$$

Comparison of Eqs. 38 and 39 shows that

$$E = mc^2 = \gamma_u m_0 c^2. \tag{40}$$

This is a very well known result.

Conservation of the three spatial components of the four-momentum in a particular inertial reference system implies conservation of the fourth component in the same reference system. This is because the length in four-space of the four-momentum is the invariant $m_0 c$. Thus, in relativistic particle mechanics, conservation of momentum implies conservation of energy.

Just as momentum and energy were combined into a contravariant four-vector to satisfy the requirements of special relativity, the force and power defined in Eqs. 37 and 39 may be combined into a four-vector force. The force as defined in Eq. 37 is not the spatial part of a four-vector. However, it does become the spatial part of a four-vector when multiplied by the factor γ_u. The contravariant four-vector force thus generated is known as the Minkowski force and may be written as

$$\mathbf{F}^i = (\gamma_u \mathbf{F}, \gamma_u P/c). \tag{41}$$

The laws of transformation from one inertial frame to another which force and power obey may be deduced from the four-vector character of $\mathbf{F}^i$. Suppose that the primed reference system in Fig. 1 moves along the positive x-axis of the laboratory inertial reference frame with the uniform velocity $\mathbf{v}$. Force and power in the primed reference system are then related to force and power in the laboratory by the transformations.

$$\gamma_{u'} F_x' = \gamma_v \gamma_u (F_x - vP/c^2) \tag{42}$$

$$\gamma_{u'} F_y' = \gamma_u F_y \tag{43}$$

$$\gamma_{u'} F_z' = \gamma_u F_z \tag{44}$$

$$\gamma_{u'} P' = \gamma_v \gamma_u (P - vF_x). \tag{45}$$

The quantities in these transformations are defined by the relationships following Fig. 1, by Eq. 10, and by the definitions

$$\mathbf{F} = d(m\mathbf{u})/dt \tag{46a}$$

$$P = d(mc^2)/dt \tag{46b}$$

$$\mathbf{F}' = d(m'\mathbf{u}')/dt' \tag{46c}$$

$$P' = d(m'c^2)/dt'. \tag{46d}$$

Equations 42 through 45 represent a true relativistic transformation in the sense that they can be solved or inverted by interchanging the primed and unprimed quantities, noting that $\mathbf{v}' = -\mathbf{v}$. The inverted set of equations may be substituted back into the original set without contradiction.

All forces of any origin whatsoever which act upon the particle in question must obey the law of transformation expressed in Eqs. 42 through 45. The equation of dynamic equilibrium of the particle requires that any applied force regardless of its nature be balanced by an inertial force. The special theory of relativity requires that this equation of equilibrium remain invariant in form and meaning for all inertial observers. Therefore, any applied force must transform in exactly the same way as an inertial force.

Thus far everything that has been said in this section applies quite rigorously to particles which have zero extension and which are characterized by an invariant property called the rest mass. This rest mass doesn't change without atomic fission of the particle. Let us consider further the problem of a single particle which moves relative to both the primed and laboratory inertial coordinate systems of Fig. 1. The energy of the particle in the laboratory coordinate system is given by

$$E = \gamma_u m_0 c^2 = mc^2 \tag{47}$$

while the energy of the same particle in the primed reference system is

$$E' = \gamma_{u'} m_0 c^2 = m'c^2. \tag{48}$$

Combining these two relationships

$$E = \gamma_u E'/\gamma_{u'} = \gamma_u m'c^2/\gamma_{u'}. \tag{49}$$

Since the primed reference system moves along the positive x-axis of the laboratory coordinates with the uniform velocity $\mathbf{v}$, it can be shown that

$$\gamma_u/\gamma_{u'} = \gamma_v(1 + vu_x'/c^2). \tag{50}$$

Equation 50 can be proved through application of the formulas for the relativistic addition of velocities (1, pp. 257–260). Substituting identity (50) into (49) yields

$$E = \gamma_v(1 + vu_x'/c^2)m'c^2. \tag{51}$$

Equation 51 gives the total energy of the particle as seen by a laboratory observer.

Next consider a system of particles which may collide with one another but which are not otherwise interacting. The total energy of such a system is merely the sum of the individual particle energies. Let m_k' denote the apparent primed frame mass of the k^{th} particle and u_{xk}' represent the x-component of velocity of the k^{th} particle relative to the primed reference frame. The total energy E_s of all the particles may be written as

$$E_s = \gamma_v c^2(\sum_k m_k' + [v/c^2] \sum_k m_k' u_{xk}'). \tag{52}$$

The second summation on the right hand side of Eq. 52 represents the net x-directed component of momentum of the system of particles relative to the primed coordinates. After considering forces of interaction, it will be attempted to treat the system of particles as a body momentarily at rest in the primed coordinate system. This would not make sense unless the net x-directed component of momentum of the system of particles relative to the primed coordinates was zero at the moment in question. Therefore, assume that

$$\sum_k m_k' u_{xk}' = 0. \tag{53}$$

The fact that the primed coordinates move along the x-axis of the laboratory coordinates is purely the result of an arbitrary choice. If the system of particles under discussion is ever to be regarded as a body at rest in the primed reference frame, the y- and z-directed components of the momentum of the system relative to the primed reference frame must also vanish. Therefore, assume that

$$\sum_k m_k' u_{yk}' = 0 \tag{54}$$

and

$$\sum_k m_k' u_{zk}' = 0. \tag{55}$$

If Eqs. 53, 54 and 55 are satisfied the total energy of the system of particles may be written without regard to the direction of motion as

$$E_s = \gamma_v c^2 \sum_k m_k'. \tag{56}$$

Note carefully that the individual particles of the system may be in motion relative to the primed reference frame. The quantity $\sum_k m_k'$ can be a function of time since the kinetic energy of the particles relative to the primed frame may change. The summation $\sum_k m_k'$ behaves like a time varying rest mass for the complete system of particles.

Similar reasoning leads to similar deductions as far as the net momentum of a system of colliding, but otherwise non-interacting

particles is concerned. Consider the components of the linear three dimensional momentum of a single particle. In the laboratory frame of reference they may be written as

$$p_x = \gamma_u m_0 u_x \tag{57a}$$

$$p_y = \gamma_u m_0 u_y \tag{57b}$$

$$p_z = \gamma_u m_0 u_z. \tag{57c}$$

In the primed frame of reference the corresponding components of momentum are

$$p_x' = \gamma_{u'} m_0 u_x' \tag{58a}$$

$$p_y' = \gamma_{u'} m_0 u_y' \tag{58b}$$

$$p_z' = \gamma_{u'} m_0 u_z'. \tag{58c}$$

Combining Eqs. 57*a* and 58*a* one obtains

$$p_x = \gamma_u u_x p_x'/\gamma_{u'} u_x'. \tag{59}$$

The ratio $\gamma_u/\gamma_{u'}$ is given in Eq. 50. If $\mathbf{v}$ is in the x-direction, the formulas for relativistic addition of velocities require that

$$u_x/u_x' = (1 + v/u_x')/(1 + vu_x'/c^2). \tag{60}$$

Substitution of Eqs. 50 and 60 into Eq. 59 yields the following expression for the x-component of momentum of a single particle as reported by a laboratory observer

$$p_x = \gamma_v(1 + v/u_x')m'u_x'. \tag{61}$$

Next let us sum the x-components of momentum of all k-particles in the system under consideration. The subscript s will be used to denote a total or system quantity.

$$p_{xs} = \gamma_v \sum_k m_k' u_{xk}' + \gamma_v v \sum_k m_k'. \tag{62}$$

The first summation on the right hand side of Eq. 62 vanishes according to the earlier assumption of Eq. 53. The remaining term is

$$p_{xs} = \gamma_v \mathbf{v} \sum_k m_k' \tag{63}$$

where $\mathbf{v}$ is in the x-direction and $\sum_k m_k'$ is the total relativistic mass of all the particles involved referred to the primed reference frame. This summation may be treated as a time-varying rest mass for the system not only as far as the system energy is concerned but also as far as the

parallel component of total momentum is concerned. Only the components of momentum perpendicular to $\mathbf{v}$ remain to be examined. It is necessary to consider only one of them, say the y-component, since the other perpendicular component must behave in exactly the same fashion. Combining Eqs. 57*b* and 58*b* one obtains the y-component of the laboratory momentum of a single particle.

$$p_y = \gamma_u u_y p_y'/\gamma_{u'} u_y'. \tag{64}$$

The relativistic addition of velocities requires that

$$u_y/u_y' = 1/\gamma_v(1 + vu_z'/c^2). \tag{65}$$

Substituting Eqs. 50 and 65 into Eq. 64 shows that the y-component of momentum of the particle is the same in the laboratory coordinate system as in the primed reference frame.

$$p_y = p_y' = m'u_y'. \tag{66}$$

Summing the y-components of the momentum of each particle yields

$$p_{sy} = \sum_k m_k' u_{yk}' = 0. \tag{67}$$

The total y-component of momentum vanishes by virtue of the earlier assumption of Eq. 54. Similarly the total z-component of momentum of the system of k-particles vanishes.

The results thus far make good sense. If a system of k-particles rattle about in a massless box and the box is momentarily at rest in the primed reference frame; Eqs. 53, 54 and 55 are satisfied. The total energy and momentum of the complete system of particles in the imaginary box may be written as

$$E_s = \gamma_v M_0 c^2 \quad \text{and} \quad \mathbf{p}_s = \gamma_v M_0 \mathbf{v} \tag{68}$$

where

$$M_0 = \sum_k m_k'. \tag{69}$$

The velocity $\mathbf{v}$ is the instantaneous velocity of the imaginary box relative to the laboratory observer and M_0 is a time-varying quantity which clearly behaves like the rest mass of the entire system of particles. In other words, M_0 is a measure of the energy content of the entire system of particles in a frame of reference in which the system has no resultant momentum. If the box full of particles is somehow heated each particle will bounce about more vigorously. From the standpoint of a laboratory observer looking at the system of particles as a whole, the increase in kinetic energy of the individual particles may be regarded as an increase in the rest mass or rest energy of the system.

Forces of interaction between the particles, other than through collision, have not yet been considered. A bit of speculation and postu-

lation seems to be necessary in order to cope with them. Imagine the system of particles under discussion to be somehow bound together into a body. In addition to the individual particle energies the system would have a certain binding energy. In the primed frame of reference, the one in which the system is momentarily at rest, let the binding energy be denoted by E_b'. This binding energy represents the work that would have to be done by a primed frame observer to overcome all the forces of attraction between particles and thus to separate the body into a system of independent particles. As the body is polarized or mechanically deformed the binding energy will change. The total energy of the body as reported by a primed frame observer is

$$E_s' = \sum_k m_k' c^2 + E_b'. \tag{70}$$

It will be assumed that mass and energy are equivalent in a completely general sense. The rest mass of the body now involves the binding energy.

$$M_{ob} = \sum_k m_k' + E_b'/c^2. \tag{71}$$

Furthermore, it will be assumed or postulated that all forms of energy transform according to the same law. Therefore, the total laboratory energy of the body may be written as

$$E_s = \gamma_v E_s' = \gamma_v M_{ob} c^2, \tag{72}$$

where M_{ob} is given in Eq. 71. Similarly it seems reasonable to continue to define the net momentum of the system as in Eq. 68 but with the rest mass as defined in Eq. 71. Thus

$$\mathbf{p}_s = \gamma_v M_{ob} \mathbf{v}. \tag{73}$$

Equations 71, 72 and 73 summarize the point of view which will be used throughout the remainder of this paper. A body will be regarded as at rest in the frame of reference in which its net momentum is zero. The total energy of the body in this zero-momentum or proper frame will be referred to as the rest energy, a quantity which may vary with time. As the body moves relative to an observer its total energy and net momentum vary with velocity in exactly the same manner as for a particle having an invariant rest mass. The preceding discussion is not intended as a proof of anything. The point of view taken represents an interesting set of assumptions or postulates. The chief arguments in favor of these assumptions have not yet been presented.

The resultant force acting on a body as a whole will be defined as the rate of change of the net momentum of the body. The total power supplied to the body will be defined as the rate of increase of the body's total energy. Suppose that a body moves with velocity $\mathbf{u}$ relative to the laboratory inertial coordinate system of Fig. 1 and with velocity $\mathbf{u}'$

relative to the primed reference frame. The proper frame of the body is the one in which its net momentum is zero and the velocities of the body are to be regarded as measured from this proper frame. The laboratory force and power definitions are

$$\mathbf{F} = d(\gamma_u M_{ob}\mathbf{u})/dt \tag{74a}$$

and

$$P = d(\gamma_u M_{ob}c^2)/dt \tag{74b}$$

while the primed frame force and power definitions are

$$\mathbf{F}' = d(\gamma_{u'} M_{ob}\mathbf{u}')/dt' \tag{74c}$$

and

$$P' = d(\gamma_{u'} M_{ob}c^2)/dt'. \tag{74d}$$

Equations 74 are identical in form with Eq. 46. However, M_{ob} is a function of time while m_0 is an invariant. This added complication leads one to wonder whether the forces and powers of Eqs. 74 transform according to the same laws as those of Eqs. 46. The answer is that they do! It can be proved that the set of transformations, Eqs. 42 through 45, remain valid even when the rest mass is time varying. In other words, the covariance of relativistic mechanics is not destroyed by postulation of the time varying rest mass concept for material bodies. Everything that has been assumed or postulated thus far is consistent with the requirements of special relativity and seems to make physical sense.

There is one readily apparent consequence of a variable rest mass. The law of conservation of energy does not require that the rate of increase of the total energy of a body be equal to the rate at which the resultant force acting on the body as a whole does work to move the body. That is, Eq. 38 or 39 is not valid if the rest mass does not remain constant. Differentiation of Eqs. 74*a* and *b* shows that

$$P = \mathbf{u}\cdot\mathbf{F} + d(M_{ob}c^2)/\gamma_u dt. \tag{75}$$

The total power supplied to a body is made up of two parts. The first part, $\mathbf{u}\cdot\mathbf{F}$, is the rate at which work is done by the resultant force to move the body as a whole relative to the laboratory observer. The second part involves the rate of increase of the rest energy. This term represents the apparent power, as seen by a laboratory observer, required to distort the body or move various parts of it relative to the frame of reference in which the resultant momentum of the body is zero.

As before the subscript zero will be used to denote quantities which are seen by an observer in the proper frame of a body. A proper time interval dt_0 appears dilated to a laboratory observer so that

$$dt = \gamma_u dt_0. \tag{76}$$

Consequently, Eq. 75 may be written as

$$P = \mathbf{u}\cdot\mathbf{F} + (1 - u^2/c^2)P_0 \tag{77}$$

where

$$P_0 = d(M_{ob}c^2)/dt_0. \tag{78}$$

Equation 77 is valid either for a body or for a particle if one recognizes that P_0 vanishes for a particle. Therefore, this relationship may be substituted into the force and power transformations, Eqs. 42 through 45, without loss of generality. In the special case in which the primed frame is instantaneously a proper one with respect to a body, the transformations take on the simpler form

$$F_{xo} = F_x - uP_0/c^2 \tag{79}$$

$$F_{yo} = \gamma_u F_y \tag{80}$$

$$F_{zo} = \gamma_u F_z \tag{81}$$

$$P_0 = P_0. \tag{82}$$

For a particle, the parallel component of force in any inertial frame is the same as the parallel component of force in the proper frame, since $P_0 = 0$. On the other hand, the parallel component of resultant force acting on a body as a whole does not transform invariantly.

Note that in Eqs. 79 through 82 the symmetry of the transformations has been destroyed by specialization to a proper frame. They can no longer be inverted or solved merely by interchanging primed (proper) and unprimed (laboratory) quantities.

F. A BODY IN MOTION UNDER THE INFLUENCE OF MECHANICAL AND ELECTROMAGNETIC FORCES

Armed with the point of view of the previous section it is now possible to demonstrate that the concept of a variable rest mass is consistent with macroscopic electrodynamics. Furthermore, the interpretations given to terms in the modified Poynting theorem of Eq. 33 can now be justified.

Consider a body in motion under the combined influence of mechanical and electromagnetic forces. More specifically, consider a grain of material containing many electric and magnetic dipoles but small enough to be of differential dimensions in a macroscopic field theory. Let $\mathbf{u}$ denote the velocity of the grain relative to an inertial observer. The equation of equilibrium of the entire grain, when dot multiplied by its velocity, is

$$\mathbf{u}\cdot{}^{e}\mathbf{F} + \mathbf{u}\cdot{}^{m}\mathbf{F} = \mathbf{u}\cdot{}^{i}\mathbf{F}. \tag{83}$$

The presuperscript m denotes a force of mechanical origin while i represents an inertial force. As before e stands for an electromagnetic

quantity. Equation 83 requires that the net applied force in the direction of $\mathbf{u}$ be balanced by an inertial reaction force in the opposite direction. Each of these types of force must transform to the proper frame of the grain according to the same law, expressed in Eq. 79. Therefore, transforming Eq. 83 to the proper frame one obtains

$$\mathbf{u}\cdot{}^{e}\mathbf{F}_0 + u^2\,{}^{e}P_0/c^2 + \mathbf{u}\cdot{}^{m}\mathbf{F}_0 + u^2\,{}^{m}P_0/c^2 = \mathbf{u}\cdot{}^{i}\mathbf{F}_0 + u^2\,{}^{i}P_0/c^2. \quad (84)$$

The equation of dynamic equilibrium of the complete grain must be preserved in form and meaning in the proper frame so that

$$\mathbf{u}\cdot{}^{e}\mathbf{F}_0 + \mathbf{u}\cdot{}^{m}\mathbf{F}_0 = \mathbf{u}\cdot{}^{i}\mathbf{F}_0. \quad (85)$$

Consequently it is necessary that power be conserved in the proper frame

$${}^{e}P_0 + {}^{m}P_0 = {}^{i}P_0 = d(M_{0}c^2)/dt_0. \quad (86)$$

The electromagnetic proper frame power, associated with heating or polarizing the grain, combined with the mechanical proper frame power, associated with any mechanical deformation of the grain, accounts for the rate of increase of the rest energy of the grain. If the rest energy of the grain was constant, the law of conservation of energy in the proper frame of the grain would be violated.

It should be pointed out that the proper frame of the grain as a whole from an electromagnetic standpoint must be the same inertial coordinate system as the proper frame of the entire grain from a mechanical standpoint. This sheds some light on the meaning of a proper frame in macroscopic electrodynamics.

It can be directly demonstrated that electromagnetic force and power transform according to the general laws, Eqs. 42 through 45. Furthermore, they satisfy Eq. 77 so that they also obey Eqs. 79 through 82. For example, it can readily be shown, in agreement with Eq. 79, that

$$\mathbf{u}\cdot{}^{e}\mathbf{F} = \mathbf{u}\cdot{}^{e}\mathbf{F}_0 + u^2\,{}^{e}P_0/c^2. \quad (87)$$

The quantities ${}^{e}\mathbf{F}_0$ and ${}^{e}P_0$ are defined in terms of the field and material vectors by Eqs. 29, 30 and 32 while the resultant laboratory electromagnetic force on the grain is given by

$${}^{e}\mathbf{F} = \int[\mathbf{E}(\rho + \rho_p) + (\mathbf{J} + \mathbf{J}_p) \times \mu_0\mathbf{H} + \mathbf{H}\rho^* - \mathbf{J}^* \times \epsilon_0\mathbf{E}]dV. \quad (88)$$

The point is that macroscopic electrodynamics as modified by Chu obeys all the laws of transformation and conservation which have been assumed or postulated in this paper for macroscopic grains of material. In other words macroscopic electrodynamics is consistent with a macroscopic mechanics which assumes a variable rest mass.

The forces acting on the grain of material in question are all resultant

forces regardless of origin. They are forces which tend to move the grain as a whole relative to a laboratory observer. The detailed forces acting on individual particles within a grain, which tend to move parts of the grain relative to its proper frame without moving the grain as a whole, show up only in the power terms and not in the resultant force terms of the transformations for force and power. Those effects of forces which are not included in the power terms must be accounted for in the resultant force terms. Similarly, those effects of forces which are not accounted for by the motion of a body as a whole must be included in the power terms. Whether a particular effect of a force is to be included in the force terms or the power terms depends on whether one wishes to consider indivisible particles, macroscopic but differential grains made up of many particles, or complete large scale bodies.

The preceding paragraph sheds much light on the meaning of the macroscopic electromagnetic force density postulated by Chu. The proper frame macroscopic electromagnetic power density of Eq. 29 accounts for those effects of electromagnetic forces associated with polarization of electric and magnetic dipoles and with heating a conductor by causing electrons to collide with atoms. All other effects of electromagnetic forces must be included in the force density of Eq. 30. Equation 30 represents a force density averaged over a large number of dipoles or particles within a macroscopic differential volume. It does not include the average stresses within dipoles or the average forces of interaction between conduction electrons and material atoms.

Consider next the conversion of electromechanical energy when a differential grain of material moves relative to a laboratory observer under the combined influence of mechanical and electromagnetic forces. The power converted to mechanical form and extracted from the system by the mechanical force source, as reported by a laboratory observer is given by

$$-\mathbf{u}\cdot{}^{m}\mathbf{F} = \mathbf{u}\cdot({}^{e}\mathbf{F} - {}^{i}\mathbf{F}). \tag{89}$$

Transforming the forces on the right hand side to the proper frame of the grain gives, for the power converted

$$-\mathbf{u}\cdot{}^{m}\mathbf{F} = \mathbf{u}\cdot[{}^{e}\mathbf{F}_0 - {}^{i}\mathbf{F}_0 + \mathbf{u}({}^{e}P_0 - {}^{i}P_0)/c^2]. \tag{90}$$

If the grain is mechanically rigid in the proper frame, the proper mechanical force cannot distort it and hence can store no energy in it. Therefore ${}^{m}P_0$ is zero. The fact that the grain is mechanically rigid in the proper frame does not mean that it can't be electromagnetically polarized or heated. Equation 86 shows that ${}^{e}P_0$ is equal to ${}^{i}P_0$ when ${}^{m}P_0$ is zero. Thus, for a rigid grain the power converted to mechanical form and extracted from the system as seen by a laboratory observer becomes simply

$$-\mathbf{u}\cdot{}^{m}\mathbf{F} = \mathbf{u}\cdot({}^{e}\mathbf{F}_0 - {}^{i}\mathbf{F}_0). \tag{91}$$

The proper frame inertial force vanishes if the rigid grain in question moves at a constant velocity relative to the laboratory. On the other hand the laboratory frame inertial force does not vanish under conditions of constant velocity. This is because the rest mass of the grain changes as it is polarized or heated electromagnetically. For constant velocity ${}^{\circ}\mathbf{F}_0$ is zero but

$$ {}^{\circ}\mathbf{F} = d(\gamma_u M_{os}\mathbf{u})/dt = (\mathbf{u}/c^2)d(M_{os}c^2)/dt_0. \tag{92} $$

Consequently, under the two restrictions of rigidity and constant velocity the mechanical power extracted from the system as seen by a laboratory observer is

$$ -\mathbf{u}\cdot{}^{m}\mathbf{F} = \mathbf{u}\cdot{}^{\circ}\mathbf{F}_0 = \mathbf{u}\cdot{}^{\circ}\mathbf{F} - u^2\,{}^{\circ}P_0/c^2. \tag{93} $$

This justifies the interpretation given to $\mathbf{u}\cdot{}^{\circ}\mathbf{F}_0$ in the modified Poynting theorem, Eq. 33.

The total electromagnetic power supplied to the moving grain as seen from the laboratory is shown by Eq. 77 to be

$$ {}^{\circ}P = \mathbf{u}\cdot{}^{\circ}\mathbf{F} + (1 - u^2/c^2)\,{}^{\circ}P_0. \tag{94} $$

Transforming the force term in Eq. 94 to the proper frame by means of Eq. 87 and simplifying

$$ {}^{\circ}P = \mathbf{u}\cdot{}^{\circ}\mathbf{F}_0 + {}^{\circ}P_0. \tag{95} $$

The quantity ${}^{\circ}P$ is the total power supplied to the grain electromagnetically as seen by the laboratory observer. It has been shown that under certain conditions $\mathbf{u}\cdot{}^{\circ}\mathbf{F}_0$ is the power mechanically extracted from the system as reported by a laboratory observer. Therefore, under the same conditions, the term ${}^{\circ}P_0$ has to represent the total electromagnetic power absorbed by the grain as reported by a laboratory observer. This justifies the interpretation given to ${}^{\circ}P_0$ in the modified Poynting theorem.

G. THE APPARENT POWER TO INCREASE REST ENERGY

In Eq. 77 it was shown that the total power supplied to a body may be divided into two reasonable parts. The first part is the power supplied by the net applied force to move the body as a whole. This is equivalent to the power supplied to increase the net momentum of the body, since the body by definition has no net momentum in its proper frame. The remainder of the total power is the apparent power, as seen by a laboratory observer, supplied to move various parts of the body relative to its rest frame. This is the power which apparently increases the rest energy of the body. The apparent power required to increase the rest energy of a body is $(1 - u^2/c^2)$ times the proper

frame power required to the increase the rest energy. Even though a laboratory and a proper frame observer watch the same physical phenomenon they need not agree as to the power involved.

The meaning of this last statement can be made clearer by an illustrative example. Consider a dipole made up of charges $+q$ and $-q$ and separated by a distance $\mathbf{d}_0$ in a proper frame. The proper dipole moment is $\mathbf{p}_0 = q\mathbf{d}_0$. The proper power expended to polarize the dipole is

$$^{\bullet}P_0 = \mathbf{E}_0 \cdot d\mathbf{p}_0/dt_0 = q\mathbf{E}_0 \cdot d\mathbf{d}_0/dt_0. \tag{96}$$

The Lorentz transformation enables one to relate this proper frame power to the power reported by a laboratory observer watching the same physical phenomenon. Separating $\mathbf{E}_0$ and $\mathbf{d}_0$ into components parallel and perpendicular to the velocity $\mathbf{u}$ of the proper frame, transforming the components to the laboratory coordinate system, and recombining leads to the result

$$(\mathbf{E} + \mathbf{u} \times \mu_0\mathbf{H}) \cdot d(q\mathbf{d})/dt = (1 - u^2/c^2)\mathbf{E}_0 \cdot d(q\mathbf{d}_0)/dt_0. \tag{97}$$

The quantity $(\mathbf{E} + \mathbf{u} \times \mu_0\mathbf{H})$ is the effective force per unit charge which the laboratory observer thinks is acting on the moving dipole. It is the Lorentz force in vacuum. The quantity $d(q\mathbf{d})/dt$ is the apparent polarization current seen by the laboratory observer. Therefore, the dot product of these two terms represents the power which the laboratory observer thinks is required to polarize the dipole. The apparent laboratory power to polarize the dipole is $(1 - u^2/c^2)$ times the proper frame power to polarize the same dipole.

H. THE POYNTING THEOREM IN TERMS OF LABORATORY QUANTITIES ALONE

The Poynting theorem of Eq. 33 may be reinterpreted in the light of the preceding discussion. Adding and subtracting the term $(u^2/c^2)\ ^{\bullet}P_0$, which is a component of laboratory power absorbed by the grain of material due to an increase in its momentum, one obtains

$$-\operatorname{div}(\mathbf{E} \times \mathbf{H})dV = (\partial w/\partial t)dV + (1 - u^2/c^2)\ ^{\bullet}P_0 + \mathbf{u} \cdot {}^{\bullet}\mathbf{F}_0 + u^2\ ^{\bullet}P_0/c^2. \tag{98}$$

The term $(1 - u^2/c^2)\ ^{\bullet}P_0$ is the apparent laboratory power absorbed by the grain of material as Joule heat and as polarization energy. It does not include any power absorbed by the grain due to increased momentum.

The two terms $\mathbf{u} \cdot {}^{\bullet}\mathbf{F}_0$ and $(u^2/c^2)\ ^{\bullet}P_0$ may be combined into the single term $\mathbf{u} \cdot {}^{\bullet}\mathbf{F}$ as shown by Eq. 87. Therefore these two terms represent that portion of the total electromagnetic power supplied to the grain which tends to move the grain as a whole. Part of this power can be extracted by a mechanical force source and the remainder increases the momentum of the grain as seen by a laboratory observer.

The new interpretations given in the last two paragraphs are valid even though the grain accelerates and deforms. In this sense they are more general than the earlier interpretations. It is important to note that $\mathbf{u}\cdot{}^{\bullet}\mathbf{F}$ never represents the power extracted from the system by a mechanical force source.

The theorem expressed in Eq. 98 can be reformulated in terms of laboratory quantities alone. Let the subscript e denote effective quantities as reported by a laboratory observer. The effective field intensities and current densities are defined as

Effective electric field intensity

$$\mathbf{E}_e = \mathbf{E} + \mathbf{u} \times \mu_0\mathbf{H} \tag{99}$$

Effective magnetic field intensity

$$\mathbf{H}_e = \mathbf{H} - \mathbf{u} \times \epsilon_0\mathbf{E} \tag{100}$$

Effective conduction current density

$$\mathbf{J}_e = \mathbf{J} - \rho\mathbf{u} \tag{101}$$

Effective electric polarization current density

$$\mathbf{J}_{pe} = \mathbf{J}_p - \rho_p\mathbf{u} \tag{102}$$

Effective magnetic polarization current density

$$\mathbf{J}_e^* = \mathbf{J}^* - \rho^*\mathbf{u}. \tag{103}$$

The laboratory current and charge densities are defined in Eqs. 15 through 18. The effective quantities represent the field intensities and current densities which a laboratory observer thinks must exist on a grain of material whose proper frame moves with velocity $\mathbf{u}$ relative to him. They are effective quantities in the same sense that the Lorentz force is an effective force.

The apparent electromagnetic power to increase the rest mass of the grain as seen from the laboratory may readily be expressed in terms of the effective quantities as

$$(1 - u^2/c^2)\,{}^{\bullet}P_0 = (\mathbf{E}_e\cdot\mathbf{J}_e + \mathbf{E}_e\cdot\mathbf{J}_{pe} + \mathbf{H}_e\cdot\mathbf{J}_e^*)dV. \tag{104}$$

The final result in terms of laboratory quantities alone is the form of the Poynting theorem given by Chu (4).

$$-\operatorname{div}(\mathbf{E} \times \mathbf{H})dV = (\partial w/\partial t)dV + (\mathbf{E}_e\cdot\mathbf{J}_e + \mathbf{E}_e\cdot\mathbf{J}_{pe} + \mathbf{H}_e\cdot\mathbf{J}_e^*)dV + \mathbf{u}\cdot{}^{\bullet}\mathbf{F} \tag{105}$$

where ${}^{\bullet}\mathbf{F}$ is defined in Eq. 88.

In words, theorem (105) says that the net electromagnetic power flowing into the region occupied by the moving grain of material at a particular instant accounts for:

1. The rate of increase of electromagnetic field energy stored in vacuum in the region occupied by the grain.
2. The apparent electromagnetic power absorbed by the grain which increases its rest energy. This energy is partly Joule heat, partly electric polarization energy, and partly magnetic polarization energy.
3. The electromagnetic power which tends to move the grain as a whole. Not all of this power can be extracted mechanically if the rest energy of the grain increases.

Equation 105 can be shown to be identical with Eq. 27. Thus the Poynting theorem has been partially transformed to a proper frame and then transformed back to the laboratory without inconsistency.

I. CONCLUSIONS

The equations and postulates of Chu have been used to show that a self consistent formulation of electrodynamics leads to an intelligible Poynting theorem even in the presence of moving media. In the process a number of points have been brought out.

1. Maxwell's macroscopic equations in their usual form are a degenerate, proper frame case of more general laws.
2. A law of physics can be regarded as invariant in form and meaning for all inertial observers only if it correctly represents the experimental facts in a non-proper frame of reference. A proper frame and a non-proper frame can not be regarded as entirely equivalent in principle according to special relativity.
3. Macroscopic electrodynamics is consistent with an extension of relativistic particle mechanics. This extension of relativistic mechanics involves some assumptions and postulates which may be far reaching in their implications. The approach suggested does not destroy the covariance of physical laws. The essentials of the approach are:

(*a*) An extended object possesses zero net linear momentum at each instant in some frame of reference called the instantaneous proper frame.

(*b*) The total energy of the object in its proper frame is the variable rest energy of the object.

(*c*) The total energy and net linear momentum in some non-proper inertial frame are related to the proper frame quantities in the same way as for a point mass.

(*d*) The resultant translational force acting on an extended object is the time rate of change of net linear momentum.

(*e*) The total power supplied to an extended object is the time rate of increase of the object's total energy.

(*f*) The resultant translational force and total power transform according to the contravariant four-vector.

$$F^i = (\gamma_u F, \gamma_u P/c). \tag{106}$$

All forces of any origin whatsoever obey this law of transformation.

(*g*) The total power is made up of two components, a part associated with the linear acceleration of the object as a whole and a part associated with the movement of portions of the object relative to its proper frame.

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OBLATENESS CORRECTION TO IMPACT POINTS OF BALLISTIC ROCKETS *

BY

ROBERT E. ROBERSON [1]

ABSTRACT

A method is developed for finding the effect of the Earth's oblateness on the impact points of ballistic rockets. Suitable descriptive variables are chosen so that the oblateness perturbations can be obtained by quadrature, and explicit results are obtained correct to first order in an oblateness parameter; specifically, changes in range angle, a lateral displacement angle, and time of flight. The short range limiting forms of the corrections are obtained and interpreted, and an illustrative numerical example is given for a long range trajectory.

INTRODUCTION

The calculation of impact points of ballistic rockets on a spherical Earth is a straightforward and fairly well-known procedure. A method which is perhaps the easiest to visualize has been described recently by Singer and Wentworth (1).[2] They consider an elliptical trajectory in inertial space and account for Earth rotation by adding the proper component of Earth velocity at burn-out and by using the time of flight to determine which Earth point has moved under the impact point in inertial space.

The purpose of this paper is to derive corresponding results taking into account the oblateness of the Earth. Specifically, deviations from a nominal trajectory of range angle, lateral displacement, and time of flight are found. The analysis is limited to first order in a small oblateness parameter and to nominal ranges less than half-way around the Earth. However, these cases should include most cases of practical interest.

The viewpoint used by Singer and Wentworth lends itself very nicely to this consideration of oblateness effects, and is retained throughout the paper. Consideration is given the behavior of the trajectory in inertial space, the trajectory for a spherical Earth being used as a generating solution. The deviations from oblateness are calculated by a perturbation method which I have used elsewhere (2) in treating orbits about an oblate Earth. Insofar as possible, the notation of this paper is the same as that used by Singer and Wentworth.

It must be recognized that there actually are two kinds of effects of Earth's oblateness on a ballistic trajectory. One of these is purely dynamical. Given a set of initial conditions (position and inertial vector velocity), the force on the rocket and therefore its path is different

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

for the oblate Earth than if the same initial conditions existed over a spherical Earth having the same mass. It is this class of perturbations which is analyzed here. Another effect, however, is purely kinematical. If the initial conditions are given in terms of such quantities as altitude, latitude, velocity relative to the Earth (resolved in a locally level coordinate system), the same numerical values of these initial conditions mean quite different things to two observers, one on an oblate and the other on a spherical Earth. (One, for example, uses geographic and the other uses geocentric latitude.) A complete analysis of oblateness effects should also relate the form of the initial conditions which are convenient for the present analysis (and which are used in (1)) to the initial conditions which would be specified most naturally by an observer fixed on an oblate Earth. This topic is not included in any detail in the present paper, although a few comments on it in the discussion section imply that for short ranges some of the dynamical and kinematical effects are compensatory. For this reason, I feel that a more complete treatment of this question is an important generalization of the work.

The paper has been directed toward the derivation of closed analytic expressions (albeit approximate through neglect of higher order terms in the oblateness parameter) for the required deviations. Although the oblateness effect can be explored numerically with high speed digital computation, and such an approach is currently popular for problems of this type, there are times when tabular or graphical results for limited sets of parameter values are no substitute for analytical results. What is more, insight into the general behavior of the solutions more often accompanies the analytic form.

After general expressions for the deviations are found as Eqs. 34–43, they are reduced to limiting forms for the short range case. It is shown that results are what would be expected in working the conventional short-range flat Earth problem after taking account of the local effect of oblateness on the gravitational field near the launch point.

One set of illustrative numerical results is displayed as Fig. 2. The locus of deviations in latitude and longitude from the nominal impact points traced out as γ varies over $[0°, 360°]$ is shown for $\lambda_L = 0$, 30°, 60°, 90° and a nominal range angle $\Delta\phi = 30°$.

PRELIMINARIES

Following Singer and Wentworth, suppose that at the initial point the geocentric latitude is λ_L, the radial distance from the center of the Earth is R_L, the velocity magnitude is v_L, the inclination of the velocity vector to the geocentric horizontal[3] is θ, and the angle from the east counterclockwise to the directed projection of $\bar{v}_L$ on the locally horizontal plane is γ. The velocity considered here is relative to inertial space.

[3] The plane normal to the geocentric vertical.

Denote the position of the rocket by the radial distance r from the center of the Earth and by two angle variables β and α (respectively a "range angle" and a "lateral angle") shown in Fig. 1. The gross

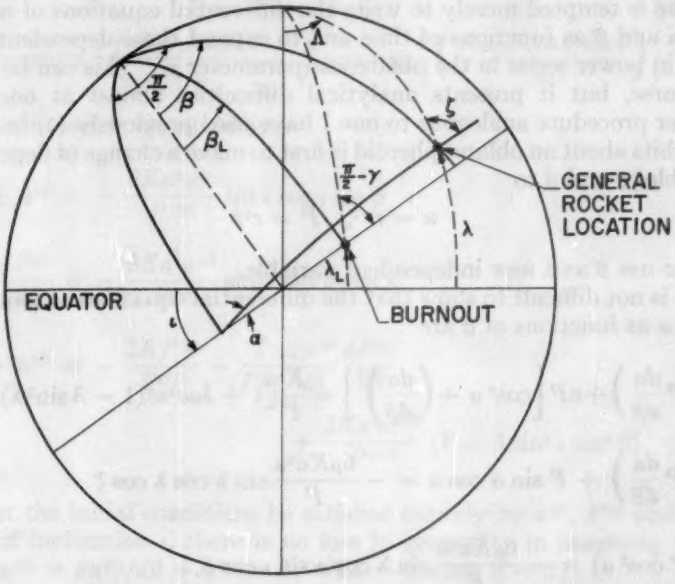

FIG. 1. Location variables and angle parameters for rocket.

motion of the rocket is described by increasing β. At the initial instant when $\lambda = \lambda_L$, we suppose that $\alpha = 0$, $\zeta = \frac{\pi}{2} - \gamma$, $\beta = \beta_L$. For a spherical Earth, α then remains zero and $\beta - \beta_L$ becomes the same as Singer and Wentworth's $\phi - \phi(R_L)$.

From the angle relationships shown in Fig. 1 it is evident that the plane from which α is measured has an inclination ι to the equatorial plane given by $\iota = \arccos(\cos\gamma \cos\lambda_L)$. The principal value is used so that $0 \leq \iota \leq \pi$. Also $\beta_L = \arcsin(\sin\lambda_L/\sin\iota)$, the principal value being used if $0 \leq \gamma \leq \pi$ and $|\lambda_L| \leq \pi/2$, and a value of β_L in the range $(\pi/2, 3\pi/2)$ if $-\pi < \gamma < 0$ and $|\lambda_L| \leq \pi/2$.

Now suppose that the potential function of the Earth is of the form

$$V = -\frac{mK}{r}\left[1 + \mu\frac{a^2}{r^2}(1 - 3\sin^2\lambda) + \cdots\right]. \tag{1}$$

Here m is the rocket mass, K is the product of the mass of the Earth by the gravitational constant, a is the equatorial radius of the Earth, and μ is a dimensionless form factor of the Earth numerically equal to 5.46×10^{-4}. The present analysis is confined to terms of the order of

μ; terms in μ^2 and the higher order terms of the potential function are neglected.

FIRST ORDER EQUATIONS

One is tempted merely to write the differential equations of motion in r, α and β as functions of time and to expand these dependent variables in power series in the oblateness parameter μ. This can be done, of course, but it presents analytical difficulties almost at once. A simpler procedure analogous to one I have used previously (2) in treating orbits about an oblate spheroid is first to make a change of dependent variables r and $\dot{\beta}$ to

$$u = r^{-1}, \quad P = r^2\dot{\beta} \tag{2}$$

and to use β as a new independent variable.

It is not difficult to show that the differential equations of motion in u, P, α as functions of β are

$$\frac{d}{d\beta}\left(P\frac{du}{d\beta}\right)+uP\left[\cos^2\alpha+\left(\frac{d\alpha}{d\beta}\right)^2\right]=\frac{K}{P}\left[1+3\mu a^2u^2(1-3\sin^2\lambda)\right] \tag{3}$$

$$\frac{d}{d\beta}\left(P\frac{d\alpha}{d\beta}\right)+P\sin\alpha\cos\alpha=-\frac{6\mu Ka^2u}{P}\sin\lambda\cos\lambda\cos\zeta \tag{4}$$

$$\frac{d}{d\beta}(P\cos^2\alpha)=-\frac{6\mu Ka^2u}{P}\sin\lambda\cos\alpha\sin\iota\cos\beta, \tag{5}$$

where

$$\left.\begin{aligned}\sin\lambda &= \sin\alpha\cos\iota+\cos\alpha\sin\iota\sin\beta\\ \cos\lambda\cos\zeta &= \cos\alpha\cos\iota-\sin\alpha\sin\iota\sin\beta\end{aligned}\right\}. \tag{6}$$

Let us now seek solutions of Eqs. 3–6 of the form

$$\left.\begin{aligned}\alpha(\beta) &= \mu\alpha^{(1)}(\beta)+0(\mu^2)\\ u(\beta) &= u^{(0)}(\beta)+\mu u^{(1)}(\beta)+0(\mu^2)\\ P(\beta) &= P^{(0)}(\beta)+\mu P^{(1)}(\beta)+0(\mu^2)\end{aligned}\right\}. \tag{7}$$

One finds, of course, that $u^{(0)}$ and $P^{(0)}$ are precisely the solutions for the spherical Earth. Specifically,[4] if $\theta\neq\pi/2$,

$$P^{(0)} = R_Lv_L\cos\theta \tag{8}$$

$$u^{(0)} = \frac{K}{(R_Lv_L\cos\theta)^2}\{1+\epsilon\cos[\beta-\beta_L+\phi(R_L)]\} \tag{9}$$

[4] If $\theta=\pi/2$, the zero order solution is degenerate and the problem must be treated differently. This special case is not pursued in the present paper.

with

$$\left.\begin{aligned}\epsilon^2 &= 1-\frac{2R_Lv_L^2}{K}\cos^2\theta+\frac{R_L^2v_L^4}{K^2}\cos^2\theta \\ \phi(R_L) &= \arccos\frac{1}{\epsilon}\left(\frac{R_Lv_L^2\cos^2\theta}{K}-1\right)=\arcsin\frac{1}{\epsilon}\left(\frac{R_Lv_L^2\cos\theta\sin\theta}{K}\right)\end{aligned}\right\}. \quad (10)$$

Now, however, we also have

$$\frac{d^2\alpha^{(1)}}{d\beta^2}+\alpha^{(1)} = -\frac{6Ka^2u^{(0)}}{P^{(0)2}}\sin\iota\cos\iota\sin\beta \quad (11)$$

$$\frac{dP^{(1)}}{d\beta} = -\frac{6Ka^2u^{(0)}}{P^{(0)}}\sin^2\iota\sin\beta\cos\beta \quad (12)$$

$$\frac{d^2u^{(1)}}{d\beta^2}+u^{(1)} = -\frac{2KP^{(1)}}{P^{(0)3}}-\frac{1}{P^{(0)}}\frac{du^{(0)}}{d\beta}\frac{dP^{(1)}}{d\beta} + \frac{3Ka^2u^{(0)2}}{P^{(0)2}}(1-3\sin^2\iota\sin^2\beta). \quad (13)$$

If we let the initial conditions be satisfied entirely by $u^{(0)}$, $P^{(0)}$ and the choice of inclination ι, there is no loss in generality in assuming that $P^{(1)} = u^{(1)} = du^{(1)}/d\beta = \alpha^{(1)} = d\alpha^{(1)}/d\beta = 0$ when $\beta = \beta_L$.

If Eqs. 8–10 are used in Eqs. 11–13, the latter can be solved in an elementary fashion to obtain $\alpha^{(1)}$, $P^{(1)}$ and $u^{(1)}$. These results in Eq. 7 give the complete lateral departure α, and the first-order corrected u and P. The latter are used later below to obtain corrected impact point and time of flight.

The general solutions of Eqs. 11–13 are the following, in which $\phi_L = \phi(R_L)$.

$$\begin{aligned}\alpha^{(1)}(\beta) = -\frac{6K^2a^2\sin\iota\cos\iota}{(R_Lv_L\cos\theta)^4}\Bigg[&\left\{\frac{1}{2}\cos\beta_L+\frac{\epsilon}{3}\cos(\beta_L-\phi_L)\right\}\sin(\beta-\beta_L) \\ &+\left\{\frac{\epsilon}{6}\sin(\beta_L+\theta_L)-\frac{\epsilon}{2}\sin(\beta_L-\phi_L)\right\}\cos(\beta-\beta_L) \\ &+\frac{\epsilon}{2}\sin(\beta_L-\phi_L)-\frac{1}{2}(\beta-\beta_L)\cos(\beta-\beta_L)\cos\beta_L \\ &+\frac{1}{2}(\beta-\beta_L)\sin(\beta-\beta_L)\sin\beta_L-\frac{\epsilon}{6}\sin 2(\beta-\beta_L)\cos(\beta_L+\phi_L) \\ &-\frac{\epsilon}{6}\cos 2(\beta-\beta_L)\sin(\beta_L+\phi_L)\Bigg] \quad (14)\end{aligned}$$

$$P^{(1)}(\beta) = -\frac{6K^2a^2\sin^2\iota}{(R_L v_L\cos\theta)^3}\Big[\frac{1}{4}\cos 2\beta_L+\frac{\epsilon}{12}\cos(2\beta_L+\phi_L)$$
$$+\frac{\epsilon}{4}\cos(2\beta_L-\phi_L)-\frac{1}{4}\cos 2\beta-\frac{\epsilon}{12}\cos(3\beta-\beta_L+\phi_L)$$
$$-\frac{\epsilon}{4}\cos(\beta+\beta_L-\phi_L)\Big]. \quad (15)$$

$$u^{(1)}(\beta) = -\frac{6K^2a^2}{(R_L v_L\cos\theta)^6}\Big[\Big\{\frac{1}{2}a_1+\frac{2}{3}a_3+\frac{3}{8}a_5+\frac{4}{15}a_7\Big\}\sin(\beta-\beta_L)$$
$$+\Big\{-a_0+\frac{1}{3}a_4+\frac{1}{8}a_6+\frac{1}{15}a_8\Big\}\cos(\beta-\beta_L)$$
$$+a_0+\frac{1}{2}a_2(\beta-\beta_L)\sin(\beta-\beta_L)-\frac{1}{2}a_1(\beta-\beta_L)\cos(\beta-\beta_L)$$
$$-\frac{1}{3}a_2\sin 2(\beta-\beta_L)-\frac{1}{3}a_4\cos 2(\beta-\beta_L)-\frac{1}{8}a_5\sin 3(\beta-\beta_L)$$
$$-\frac{1}{8}a_6\cos 3(\beta-\beta_L)-\frac{1}{15}a_7\sin 4(\beta-\beta_L)$$
$$-\frac{1}{15}a_8\cos 4(\beta-\beta_L)\Big] \quad (16)$$

in which

$$\left.\begin{aligned}
a_0 &= -\frac{1}{2}\sin^2\iota\cos 2\beta_L-\frac{\epsilon}{6}\sin^2\iota\cos(2\beta_L+\phi_L)-\frac{\epsilon}{2}\sin^2\iota\cos(2\beta_L-\phi_L)\\
&\quad -\frac{1}{16}\epsilon^2\sin^2\iota\cos(2\beta_L-2\phi_L)-\frac{1}{2}\Big(1+\frac{\epsilon^2}{2}\Big)\Big(1-\frac{3}{2}\sin^2\iota\Big)\\
a_1 &= \epsilon\Big(1-\frac{3}{2}\sin^2\iota\Big)\sin\phi_L\\
a_2 &= -\epsilon\Big(1-\frac{3}{2}\sin^2\iota\Big)\cos\phi_L\\
a_3 &= \Big(\frac{1}{4}+\frac{3\epsilon^2}{8}\Big)\sin^2\iota\sin 2\beta_L+\frac{\epsilon^2}{4}\Big(1-\frac{3}{2}\sin^2\iota\Big)\sin 2\phi_L\\
a_4 &= -\Big(\frac{1}{4}+\frac{3\epsilon^2}{8}\Big)\sin^2\iota\cos 2\beta_L-\frac{\epsilon^2}{4}\Big(1-\frac{3}{2}\sin^2\iota\Big)\cos 2\phi_L\\
a_5 &= \frac{5}{6}\epsilon\sin^2\iota\sin(2\beta_L+\phi_L)
\end{aligned}\right\}. \quad (17)$$

(*continued*)

$$a_6 = -\frac{5}{6}\,\epsilon \sin^2 \iota \cos\,(2\beta_L + \phi_L)$$

$$a_7 = \frac{5}{16}\,\epsilon^2 \sin^2 \iota \sin\,(2\beta_L + 2\phi_L)$$

$$a_8 = -\frac{5}{16}\,\epsilon^2 \sin^2 \iota \cos\,(2\beta_L + 2\phi_L)$$

It is clear from these results that it is difficult in practice, though not in principle, to proceed any further with the general analytic form; the subsequent manipulations are not prohibitively tedious, however.

IMPACT CORRECTIONS

Impact occurs at the value β_I of β which makes $r(\beta_I) = R_I$, where R_I is a preassigned radial distance. In particular, it may equal R_L as in (1), and it is assumed in the sequel that this is the case. Thus, to find β_I we need to solve

$$u^{(0)}(\beta_I) + \mu u^{(1)}(\beta_I) = 1/R_I. \tag{18}$$

Let us write the total range angle in the form

$$\Delta\phi^{(0)} + \mu\Delta\phi \tag{19}$$

where $\Delta\phi^{(0)}$ is the same as $\Delta\phi$, Singer and Wentworth's total range angle.[5] Using Eq. 19 in Eq. 18, with $\beta_I = \beta_L + \Delta\phi + \mu\Delta\phi^{(1)}$, range angle correction becomes

$$\Delta\phi^{(1)} = -\left.\frac{u^{(1)}(\beta)}{du^{(0)}/d\beta}\right|_{\beta_L+\Delta\phi} = \frac{R_I u^{(1)}(\beta_I)}{\tan\theta_I}. \tag{20}$$

The lateral angle at impact, to the same order of approximation, is simply $\mu\alpha_I^{(1)} \equiv \mu\alpha^{(1)}(\beta_L + \Delta\phi)$.

Let us now turn to the impact latitude and longitude changes. The impact latitude λ_I is given by Eq. 6 in which $\beta = \beta_I = \beta_L + \Delta\phi + \mu\Delta\phi^{(1)}$. The impact longitude Λ_I *relative to inertial space* is obtained from an analogous relationship

$$\Lambda = \arccos\,[\cos\alpha \cos\beta/\cos\lambda]. \tag{21}$$

This is obtained by inspection from Fig. 1, the longitude being measured positively toward the east from the ascending node of the trajectory. It is understood, of course, that the inverse function must be handled

[5] Or a generalization thereof if $R_I \neq R_L$. It is not a severe restriction on the present analysis to assume $R_I = R_L$, provided the two do not differ very much. If a small difference $R_I - R_L$ exists it contributes primarily to the zero order solution, giving a new value of $\Delta\phi^{(0)}$ The latter then replaces the $\Delta\phi$ used below in the formulas for the oblateness corrections.

carefully. Denoting L by the longitude *change on the Earth* between the initial point and impact,

$$L = \Lambda_I - \Lambda_L - \Omega\tau, \tag{22}$$

where Ω is the rotational angular speed of the Earth, 2π/sidereal day. Finally, the time of flight is given by

$$\tau = \int_{\beta_L}^{\beta_I} \frac{d\beta}{u^2(\beta)P(\beta)} \tag{23}$$

from Eq. 2.

Let us represent the impact latitude λ_I, the change in longitude on the Earth L, and the time of flight τ by power series in μ of the form

$$\left.\begin{aligned} \lambda_I &= \lambda_I^{(0)} + \mu\lambda_I^{(1)} + 0(\mu^2) \\ L &= L^{(0)} + \mu L^{(1)} + 0(\mu^2) \\ \tau &= \tau^{(0)} + \mu\tau^{(1)} + 0(\mu^2) \end{aligned}\right\}. \tag{24}$$

The solutions for the components of $\Delta\lambda$ are

$$\sin \lambda_I^{(0)} = \sin \iota \sin (\beta_L + \Delta\phi) \tag{25}$$

$$\lambda_I^{(1)} = \frac{\alpha_I \cos \iota + \cos (\beta_L + \Delta\phi) \sin \iota \Delta\phi^{(1)}}{\cos \lambda_I^{(0)}}. \tag{26}$$

Thus when $\Delta\phi$ is obtained from (1), $\lambda_I^{(0)}$ is obtained from Eq. 25 and $\lambda_I^{(1)}$ from Eq. 26, using $\alpha_I^{(1)}$ and $\Delta\phi^{(1)}$ found earlier above. Equation 25 can be rewritten with β_L eliminated as[6]

$$\lambda_I^{(0)} = \arcsin (\sin \Delta\phi \sin \gamma \cos \lambda_L + \cos \Delta\phi \sin \lambda_L). \tag{27}$$

Next consider τ. Equation 23 can be rewritten as

$$\tau = \int_{\beta_L}^{\beta_L + \Delta\phi + \mu\Delta\phi^{(1)}} \frac{d\beta}{u^{(0)2}P^{(0)}} - \mu \int_{\beta_L}^{\beta_L + \Delta\phi} \left[\frac{2u^{(1)}}{P^{(0)}u^{(0)3}} + \frac{P^{(1)}}{P^{(0)2}u^{(0)2}} \right] d\beta.$$

Expanding the first of these integrals as a power series in μ (retaining only the μ-term), we see that

$$\tau_0 = \int_{\beta_L}^{\beta_L + \Delta\phi} \frac{d\beta}{u^{(0)2}P^{(0)}} \tag{28}$$

$$\tau_1 = \frac{\Delta\phi^{(1)}}{\left[u^{(0)2}P^{(0)}\right]_{\beta_L + \Delta\phi}} - \int_{\beta_L}^{\beta_L + \Delta\phi} \left[\frac{2u^{(1)}}{P^{(0)}u^{(0)3}} + \frac{P^{(1)}}{P^{(0)2}u^{(0)2}} \right] d\beta. \tag{29}$$

[6] This differs from the impact latitude given in (1), the latter being incorrect except for $\lambda_L = 0$. A similar error exists in the longitude change given there.

Although it is a little messy to evaluate the integral in Eq. 29, there is no conceptual difficulty.

Now returning to Eqs. 22 and 23,

$$L = \arccos\left(\frac{\cos\beta_I}{\cos\lambda_I}\right) - \arccos\left(\frac{\cos\beta_L}{\cos\lambda_L}\right) - \Omega(\tau_0 + \mu\tau_1).$$

But

$$\frac{\cos\beta_I}{\cos\lambda_I} = \frac{\cos(\beta_L + \Delta\phi)}{\cos\lambda_I^{(0)}}\left[1 + \mu\frac{\sin\lambda_I^{(0)}\cos\iota}{\cos^2\lambda_I^{(0)}} \times \left\{\alpha_I - \frac{\tan(\beta_L + \Delta\phi)\cos\iota}{\sin\lambda_I^{(0)}}\Delta\phi^{(1)}\right\}\right] \quad (30)$$

so that

$$L^{(0)} = \arccos\left[\frac{\cos(\beta_L + \Delta\phi)}{\cos\lambda_I^{(0)}}\right] - \arccos\left(\frac{\cos\beta_L}{\cos\lambda_L}\right) - \Omega\tau_0 \quad (31)$$

and

$$L^{(1)} = \frac{1}{\cos^2\lambda_I^{(0)}}[\cos\iota\Delta\phi - \alpha_I\sin\iota\cos(\beta_L + \Delta\phi)] - \Omega\tau_1. \quad (32)$$

Eliminating β_L from Eq. 31, we have the alternative form

$$L^{(0)} = \arcsin\left(\frac{\sin\Delta\phi\cos\gamma}{\cos\lambda_I^{(0)}}\right) - \Omega\tau_0. \quad (33)$$

The essential deviations from the spherical Earth case are α_I, $\Delta\phi^{(1)}$ and τ_1. Sometimes it is convenient to assess the effects of oblateness in terms of them alone; in other cases, the resulting latitude and longitude deviations given by Eqs. 26 and 32 are more convenient.

BASIC RESULTS

In Eq. 10, v_L and θ are considered basic parameters and ϵ and $\phi(R_L)$ (or $\Delta\phi$) are derived. It is convenient for geometric visualization to consider the geometric parameters $\Delta\phi$ and θ to be given, and to eliminate v_L and ϵ from the equations. Equations 10 can be rewritten

$$\left.\begin{aligned}\frac{R_L v_L^2}{K} &= \frac{\sin\frac{\Delta\phi}{2}}{\cos\theta\sin\left(\theta + \frac{\Delta\phi}{2}\right)}\\ \epsilon &= \frac{\sin\theta}{\sin\left(\theta + \frac{\Delta\phi}{2}\right)}\end{aligned}\right\}. \quad (34)$$

The impact value α_I can be found in these terms from Eq. 14. Recognize that for the case $R_I = R_L$ the relation $\Delta\phi = 2\pi - 2\phi(R_L)$ holds, and use it to eliminate $\phi(R_L)$ in favor of $\Delta\phi$ in Eq. 14. Also put $\beta - \beta_L = \Delta\phi$ in this equation. The factor $(R_L v_L^2/K)^{-2}$ puts a term $\left(\sin\frac{\Delta\phi}{2}\right)^{-2}$ into α_I, raising some question about the behavior as $\Delta\phi \to 0$. That there is no problem from this source could be seen by converting all terms in α_I involving $\Delta\phi$ into powers of $\sin\frac{\Delta\phi}{2}$ and $\cos\frac{\Delta\phi}{2}$, rather than leaving them in terms of multiple angles as in Eq. 14. However, the following form obtained directly from Eq. 14 is perhaps the simplest.

$$\alpha_I = -6\mu\frac{a^2}{R_L^2}\cos\gamma\cos\lambda_L\left[\frac{\sin\left(\theta+\frac{\Delta\phi}{2}\right)}{\sin\frac{\Delta\phi}{2}}\right]^2 \times\left[\left\{\frac{1}{2}\Delta\phi\sin\Delta\phi+\frac{\epsilon}{2}\left(\cos\frac{3\Delta\phi}{2}-\cos\frac{\Delta\phi}{2}\right)\right\}\sin\lambda_L + \left\{\frac{1}{2}(\sin\Delta\phi-\Delta\phi\cos\Delta\phi)-\epsilon\left(\sin\frac{\Delta\phi}{2}-\frac{1}{3}\sin\frac{3\Delta\phi}{2}\right)\right\} \times\sin\gamma\cos\lambda_L\right] \quad (35)$$

in which we have used $\sin\iota\,\beta_L = \sin\lambda_L$ and $\sin\iota\cos\beta_L = \sin\gamma\cos\lambda_L$.

One can find $u_I^{(1)}$ similarly. However, it actually is $\Delta\phi^{(1)}$ that is of interest, $\Delta\phi^{(1)} = -R_L u_I^{(1)}\cot\theta$ for the case $R_I = R_L$. By properly grouping terms, it would be evident that the behavior for small $\Delta\phi$ is such that $\lim_{\Delta\phi\to 0}\Delta\phi^{(1)} = 0$, but the form of the result is rather cumbersome and it is more straightforward to evaluate the limit by expanding in a power series in $\Delta\phi$ and retaining terms through fourth order. For the same reason, Eq. 34 is not used to eliminate ϵ completely from the result at this stage. We obtain, after some manipulation,

$$\Delta\phi^{(1)} = 6\frac{a^2}{R_L^2}\frac{\sin^3\left(\theta+\frac{\Delta\phi}{2}\right)}{\sin\theta\cos^2\theta\sin^3\frac{\Delta\phi}{2}}\left[\left\{\left(-\frac{1}{4}+\frac{\epsilon^2}{6}\right)-\frac{\epsilon}{4}\Delta\phi\sin\frac{\Delta\phi}{2} + \frac{5\epsilon}{12}\cos\frac{\Delta\phi}{2}+\left(\frac{1}{6}-\frac{17}{48}\epsilon^2\right)\cos\Delta\phi-\frac{5\epsilon}{16}\cos\frac{3\Delta\phi}{2}\right.\right.$$

(*continued*)

$$+\left(\frac{1}{12}+\frac{\epsilon^2}{6}\right)\cos 2\Delta\phi-\frac{5\epsilon}{48}\cos\frac{5\Delta\phi}{2}+\frac{\epsilon^2}{48}\cos 3\Delta\phi\Big\}$$

$$-\left\{\left(\frac{1}{4}+\frac{13}{24}\epsilon^2\right)-\frac{3\epsilon}{4}\Delta\phi\sin\frac{\Delta\phi}{2}+\frac{\epsilon}{6}\cos\frac{\Delta\phi}{2}\right.$$

$$-\left(\frac{1}{3}+\frac{11}{16}\epsilon^2\right)\cos\Delta\phi-\frac{\epsilon}{16}\cos\frac{3\Delta\phi}{2}+\left(\frac{1}{12}+\frac{\epsilon^2}{8}\right)\cos 2\Delta\phi$$

$$\left.-\frac{5\epsilon}{48}\cos\frac{5\Delta\phi}{2}+\frac{\epsilon^2}{48}\cos 3\Delta\phi\right\}\cos^2\gamma\cos^2\lambda_L$$

$$+\left\{\left(1+\frac{\epsilon^2}{24}\right)-\frac{13\epsilon}{12}\cos\frac{\Delta\phi}{2}+\left(\frac{3}{8}\epsilon^2-\frac{5}{6}\right)\cos\Delta\phi\right.$$

$$+\frac{7\epsilon}{8}\cos\frac{3\Delta\phi}{2}-\left(\frac{1}{6}+\frac{3}{8}\epsilon^2\right)\cos 2\Delta\phi+\frac{5\epsilon}{24}\cos\frac{5\Delta\phi}{2}$$

$$\left.-\frac{\epsilon^2}{24}\cos 3\Delta\phi\right\}\sin^2\lambda_L+\left\{-\frac{17\epsilon}{12}\sin\frac{\Delta\phi}{2}\right.$$

$$+\left(\frac{1}{3}+\frac{5}{8}\epsilon^2\right)\sin\Delta\phi+\frac{\epsilon}{8}\sin\frac{3\Delta\phi}{2}-\left(\frac{1}{6}+\frac{\epsilon^2}{4}\right)\sin 2\Delta\phi$$

$$\left.+\frac{5\epsilon}{24}\sin\frac{5\Delta\phi}{2}-\frac{\epsilon^2}{24}\sin 3\Delta\phi\right\}\sin\gamma\sin\lambda_L\cos\lambda_L. \quad (36)$$

Again the power series expansion is useful, but it is left to the reader to carry out the details. It is worth noting that the lowest order non-vanishing term in the series is that in $\Delta\phi^4$.

The procedure for finding τ_1 is not carried through in detailed steps here, but can be outlined briefly. Into Eq. 29 substitute $P^{(1)}$ and $u^{(1)}$ from Eqs. 15 and 16. Change variables in the integral by $x = \beta - \beta_L + \phi - \pi$, so that the limits of integration become $\pm\,\Delta\phi/2$. Expand the trigonometric functions of x in resulting integrand and discard odd functions of x which vanish in the integration, rewriting the remaining integral with the range $(0, \Delta\phi/2)$. The result at this point is of the form

$$\tau_1=\frac{R_L}{v_L\cos\theta}\left[\Delta\phi^{(1)}+12\,\frac{a^2}{R_L^2}\int_0^{\Delta\phi/2}\frac{b_0+b_1\cos x+b_2\cos 2x x+b_3\sin x}{(1-\epsilon\cos x)^3}\,dx\right]. \quad (37)$$

Here

$$b_0=\left\{-\frac{1}{4}(1-\epsilon^2)+\epsilon\cos\frac{\Delta\phi}{2}-\frac{\epsilon^2}{4}\cos\Delta\phi\right\}$$

$$-\left\{\frac{3}{4}(1+\epsilon^2)+\epsilon\cos\frac{\Delta\phi}{2}-\frac{\epsilon^2}{4}\cos\Delta\phi\right\}\cos^2\gamma\cos^2\lambda_L$$

(*continued*)

$$+\left\{\frac{3}{2}-2\epsilon\cos\frac{\Delta\phi}{2}+\frac{\epsilon^2}{2}\cos\Delta\phi\right\}\sin^2\lambda_L$$

$$+\left\{-\epsilon\sin\frac{\Delta\phi}{2}+\frac{\epsilon^2}{2}\sin\Delta\phi\right\}\sin\gamma\sin\lambda_L\cos\lambda_L. \quad (38)$$

$$b_1=\left\{-\frac{3\epsilon}{4}+\left(\frac{1}{3}-\frac{\epsilon^2}{12}\right)\cos\frac{\Delta\phi}{2}-\frac{\epsilon}{4}\cos\Delta\phi+\frac{\epsilon^2}{12}\cos\frac{3\Delta\phi}{4}\right\}$$

$$+\left\{\frac{5\epsilon}{4}+\left(\frac{2}{3}+\frac{\epsilon^2}{3}\right)\cos\frac{\Delta\phi}{2}-\frac{\epsilon}{4}\cos\Delta\phi\right\}\cos^2\gamma\cos^2\lambda_L$$

$$+\left\{\epsilon-\left(\frac{5}{3}+\frac{\epsilon^2}{12}\right)\cos\frac{\Delta\phi}{2}+\epsilon\cos\Delta\phi-\frac{\epsilon^2}{4}\cos\frac{3\Delta\phi}{2}\right\}\sin^2\lambda_L$$

$$+\left\{\left(\frac{2}{3}+\frac{\epsilon^2}{3}\right)\sin\frac{\Delta\phi}{2}-\frac{\epsilon}{2}\sin\Delta\phi\right\}\sin\gamma\sin\lambda_L\cos\lambda_L \quad (39)$$

$$b_2=\left\{-\frac{\epsilon^2}{12}-\frac{1}{12}(1-\epsilon^2)\cos\Delta\phi\right\}$$

$$+\left\{\frac{\epsilon^2}{4}+\frac{1}{12}(1-\epsilon^2)\cos\Delta\phi\right\}\cos^2\gamma\cos^2\lambda_L$$

$$+\frac{1}{6}(1-\epsilon^2)\cos\Delta\phi\sin^2\lambda_L$$

$$+\frac{1}{6}(1-\epsilon^2)\sin\Delta\phi\sin\gamma\sin\lambda_L\cos\lambda_L \quad (40)$$

$$b_3=-\frac{\epsilon}{2}+\frac{3\epsilon}{2}\cos^2\gamma\cos^2\lambda_L. \quad (41)$$

By substituting $2\cos^2 x-1$ for $\cos 2x$ and properly grouping terms in the numerator of the integral of Eq. 39, this integral can be reduced to a sum of terms of the form $\int_0^{\Delta\phi/2}[1-\epsilon\cos x]^n\,dx (n=-1,-2,-3)$ and $\int_0^{\Delta\phi/2}x\sin x(1-\epsilon\cos x)^{-3}dx$ which can be handled without much difficulty.

A convenient representation in terms of a single integral, available from tables, is

$$\tau_1=\frac{R_L}{v_L\cos\theta}\left[\Delta\phi^{(1)}+12\frac{a^2}{R_L^2}\left\{-\frac{b_3\Delta\phi}{4\epsilon\left(1-\epsilon\cos\frac{\Delta\phi}{2}\right)^2}\right.\right.$$

$$+\left[\left(b_0-b_2+\frac{b_3}{2\epsilon}\right)+\left(2b_0-2b_2+\frac{b_1}{\epsilon}+\frac{b_3}{2\epsilon}\right)\epsilon\frac{\partial}{\partial\epsilon}\right.$$

$$\left.\left.\left.+\left(b_0-b_2+\frac{b_1}{\epsilon}+\frac{2b_2}{\epsilon^2}\right)\frac{\epsilon^2}{2}\frac{\partial^2}{\partial\epsilon^2}\right]\int_0^{\Delta\phi/2}\frac{dx}{1-\epsilon\cos x}\right\}\right]. \quad (42)$$

The integral in Eq. 42 is equal to

$$\frac{2}{\sqrt{1-\epsilon^2}} \arctan\left(\sqrt{\frac{1+\epsilon}{1-\epsilon}} \tan\frac{\Delta\phi}{4}\right). \tag{43}$$

The power series expansion of the integral can be used to show that τ_1 is proportional to $\Delta\phi$ for short ranges.

DISCUSSION

Now that the general form of the results has been obtained, we can take up two important questions. First, what are the practicalities of applying this analysis? Second, what form do the results take in the short range case, and how should these be interpreted?

Although the procedure outlined above can be used in a particular case to find the oblateness correction, the expressions certainly are cumbersome in their general analytic forms. It is useful to see how the problem can be reduced to more tractable size. A method for doing this exists which compounds the general case from four special cases, each of which can be handled in a somewhat simpler form.

Note that two of the three basic corrections of interest, $\Delta\phi_I^{(1)}$ and τ_1 involve coefficients which are linear combinations of the factors $\cos^2\gamma\cos^2\lambda_L$, $\sin^2\lambda_L$, $\sin\gamma\sin\lambda_L\cos\lambda_L$, and unity. Suppose that the following special cases are considered:

Case 1 Equatorial ($\lambda_L = \gamma = 0$)

Case 2 Meridian $\left(\gamma = \frac{\pi}{2}\right)$ with equatorial launch ($\lambda_L = 0$)

Case 3 Meridian with polar launch ($\lambda_L = \pi/2$)

Case 4 Meridian with $\lambda_L = \pi/4$.

Then it is clear that the general forms of the basic corrections are given by the following combinations of values:

$$\begin{aligned}\text{Case 2} &+ (\text{Case 1} - \text{Case 2})\cos^2\gamma\cos^2\lambda_L \\ &+ (\text{Case 3} - \text{Case 2})\sin^2\lambda_L \\ &+ (\text{Case 4} - \tfrac{1}{2}\,\text{Case 3} - \tfrac{1}{2}\,\text{Case 2})\sin\gamma\sin\lambda_L\cos\lambda_L. \end{aligned} \tag{44}$$

Note, though, that the combination of Eq. 44 must be made *before* using $\Delta\phi^{(1)}$ and τ_1 with α_I, to compute $\Delta\lambda_I^{(1)}$ and $L^{(1)}$.

With respect to the practicality of the method, one of the principal difficulties is the fact that α_I, $\Delta\phi^{(1)}$ and τ_1 all approach indeterminate forms at short range. This means that the analytical evaluation of their behavior is tedious: a number of terms must be carried so that some remain after the several lower order terms in $\Delta\phi$ drop out. Further, it means that numerical evaluation at short ranges must be handled with great care in order to avoid trouble from small differences.

On the other hand, the problems of numerical evaluation are much less acute at longer ranges. In any case, all the machinery is there. Results are in closed form (to first order in the oblateness parameter), which I feel is much more convenient than when a direct perturbation analysis in the time domain is attempted. In summary, numerical values of the oblateness corrections can be obtained without excessive difficulty from these results, provided only that proper precautions are taken to avoid the difficulties at short ranges mentioned above.

Another way of gaining insight into the behavior is to consider the short range case ($\Delta\phi$ small). Equation 35 gives

$$\alpha_I = -\frac{3\mu a^2 \sin 2\iota \tan\theta}{R_L^2}[\Delta\phi \sin\beta_L + \tfrac{1}{3}(\Delta\phi)^3 \cos\beta_L + \cdots]. \quad (45)$$

For typical cases the ratio $\alpha_I/\Delta\phi$ is of the order of $3\mu = 1.64 \times 10^{-3}$, which can be interpreted as an aiming error of the order of $1\frac{1}{2}$ mil. It arises from the fact that the vertical about which γ is measured does not point toward the center of curvature of the trajectory unless $\iota = 0$ or $\beta_L = 0$.

It is of considerable interest that if v_L, θ and γ were resolved into a gravitational local level coordinate system at the burn-out point, the effective γ in the latter system would be decreased by precisely $-\lim_{\Delta\phi\to 0}(\alpha_I/\Delta\phi)$. Recognizing now that the trajectory departs from the effective γ-value by the angle $\alpha_I/\Delta\phi$, it follows that an observer in the gravitational system would not see any short-range drift laterally out of the trajectory plane lying at the actual angle γ north from east in his system. On the other hand, a systematic aiming error again appears to an observer in a geographic coordinate system. Space does not permit a proof of these assertions, but it can be supplied easily by the reader.

In the short range case, we also find from Eq. 36 that

$$\Delta\phi^{(1)} = -3\frac{a^2}{R_L^2}[(1 - 3\sin^2\lambda_L + \tan\theta \sin\gamma \sin 2\lambda_L)\Delta\phi + 0(\Delta\phi^2)]. \quad (46)$$

This result can be interpreted from various points of view, of which the simplest is the following. The gravitational field strength at a given R_L is greater for the oblate Earth than for a spherical Earth of the same mass by the amount $\frac{3Ka^2}{R_L^4}(1 - 3\sin^2\lambda_L)$, from Eq. 1. Working the standard short-range projectile problem with this increased gravity, assumed constant over the trajectory, results in precisely the diminution of range given by the $(1 - 3\sin^2\lambda_L)$ terms in Eq. 46. Also, because of the oblateness there is a projection of the gravitational field vector in the plane of the trajectory equal at the burn-out point to $\sin\gamma$ times the angle between geocentric and gravitational verticals, the latter being

$6\mu \frac{a^2}{R_L^2} \sin \lambda_L \cos \lambda_L$. If this is assumed a constant force over the trajectory and the usual short range case is worked again, the remaining term in Eq. 46 is obtained. This latter term would not be apparent to an observer whose trajectory plane is chosen by a rotation γ about the gravitational vertical rather than the geocentric vertical.

The time-of-flight correction for the short range case can be obtained by expanding expression 43 in a power series in $\sqrt{(1+\epsilon)/(1-\epsilon)} \tan \frac{\Delta\phi}{4}$, retaining fifth powers, and then substituting the short-range values of ϵ, b_0, b_1, b_2, and b_3. The greatest care must be used not to inadvertently discard terms which ultimately will contribute to terms of first order in $\Delta\phi$; this means that $\Delta\phi^3$ terms must be retained in some places, $\Delta\phi^5$ terms in others. It is found that

$$\tau_1 = \frac{R_L}{v_L \cos \theta}\left[\Delta\phi^{(1)} + 3\frac{a^2}{R_L^2}\Delta\phi \tan\theta \sin 2\lambda_L \sin\gamma + 0(\Delta\phi^2)\right]$$

$$= -\frac{3a^2\Delta\phi(1 - 3\sin^2\lambda_L)}{R_L v_L \cos\theta} + 0(\Delta\phi^2). \quad (47)$$

This is identically the time decrease which would result from giving the gravitational field strength its increased value (as explained in the previous paragraph) and working the simplified problem of a short-range trajectory on a flat Earth.

We see, therefore, from Eqs. 45–47, that the general results obtained for trajectories over an oblate Earth properly reduce to the elementary flat-Earth solution at short ranges, provided appropriate adjustments are made in the gravitational field used for the latter.

ILLUSTRATIVE EXAMPLE

With five basic parameters, R_L, λ_L, θ, γ, and v_L, it is not possible to give in this paper anything approaching a complete picture of the behavior. In this illustrative example, I choose for the θ-value the "maximum-range" value $\theta = (\pi - \phi)/4$, recognizing that this value may become too small from practical considerations at the longer ranges. Next, v_L is eliminated as a basic parameter in favor of $\Delta\phi$, using Eq. 34. Now R_L enters the oblateness corrections only through the multiplicative ratios a^2/R_L^2 (in α_I and $\Delta\phi^{(1)}$) and $\sqrt{a/R_L}$ (in τ_1), which are of the order of unity for most cases of practical interest. In the examples, therefore, R_L is set equal to a without affecting the corrections very much. Naturally, in a serious calculation this assumption would not be made.

There are two other characteristics of R_L that bear mention here. First, it does determine the linear distance over the Earth corresponding

to a given $\Delta\phi$, but by taking $\Delta\phi$ itself rather than linear range as a basic parameter we need make no assumption about R_L. Second, it may not really be fair to compare results at various λ_L using the same value of R_L (as we shall do below) because altitude above the surface rather than R_L may be a more natural parameter, and comparing cases at constant altitude implies that R_L varies with λ_L on an oblate Earth.[7] It must be remembered, though, that the paper basically concerns the trajectory emanating from and terminating on a spherical shell about the Earth. Accounting for the Earth's geometrical oblateness in relating R_L and R_I to altitudes above the surface that might be specified by a ground-fixed observer is a separate problem which must be treated elsewhere.

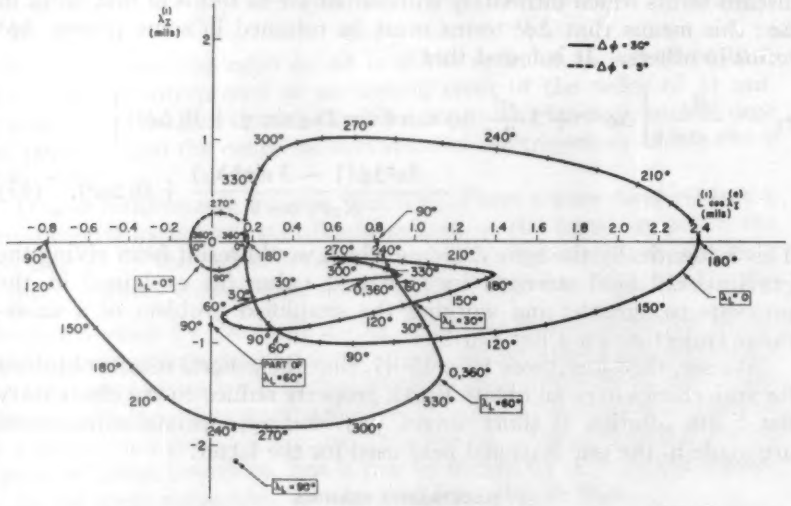

FIG. 2. Latitude and longitude deviations from aiming point as a function of launch azimuth.

It is clear that such geometrical considerations must be added to complete the theory, for they may affect the zeroth order solution of the trajectory problem by an amount comparable to the oblateness corrections derived here. It is equally clear, though, that they will primarily enter the zeroth order solution, and only as second order changes to the results of this paper. Therefore, the initial and terminal conditions as seen by a ground-fixed observer can be taken up separately without vitiating the present analysis.

From the above remarks, we now need consider only the parameters λ_L, γ, and $\Delta\phi$. A convenient representation of the oblateness effects can be obtained as follows. Suppose the range $\Delta\phi$ is fixed. Next specify a burn-out latitude λ_L and let γ vary over $(0°, 360°)$. The loci

[7] See remarks on kinematical effects of oblateness in the Introduction.

of nominal impact points is a small circle on the sphere R_L centered on the burn-out point and having (angular) radius $\Delta\phi$. At each nominal impact point, there is a deviation in latitude and longitude. Let the deviations be plotted relative to a common origin, ignoring the fact that each pertains to a different nominal impact point. Then a locus of deviations in this diagram is swept out as γ is varied, providing one kind of picture of the oblateness effects.

Figure 2 is a diagram of this kind for the nominal range $\Delta\phi = 30°$. The latitude deviation is plotted as the ordinate, but it is convenient to take the abscissa as $L^{(1)} \cos \lambda_I^{(0)}$ rather than simply $L^{(1)}$, inasmuch as the former is fundamental when one is interested in the linear distance on the sphere represented by $L^{(1)}$. Both deviations are in milliradians, so that one unit on either axis represents approximately four miles on the sphere of radius a. Four burn-out latitudes are used, $\lambda_L = 0, 30°, 60°$ $90°$, while γ is allowed to vary over $(0, 360°)$ in each case (except $\lambda_L = 90°$, where the coordinate system used here requires that $\gamma = 90°$).

To get one indication of how these characteristics may change with range, the small dashed circle in the figure represents the deviations at $\Delta\phi = 5°$, obtained from the first terms in the power series expansions of the previous section.

The rather strange appearance of the $\lambda_L = 30°$ and $60°$ curves can be explained as follows. The cusp of the $\lambda_L = 30°$ results from the fact that $L^{(1)}$ has two components, one geometric from α_I and $\Delta\phi^{(1)}$ which would be a distorted ellipse on Fig. 2, and the other a time component which shifts the points of the latter curve to the right by variable amounts. The shift increases rapidly to both sides of $\gamma = 270°$, resulting in a "tailing" of the shifted curve.

The fact that the $\lambda_L = 60°$ curve does not close (that is, there is an $L^{(1)}$ discontinuity at $\gamma = 90°$) merely reflects the facts that longitude is conventionally (in this problem) measured clockwise around the entire Earth and that the particular choice of parameters ($\lambda_L = 60°$, $\Delta\phi = 30°$) causes the locus of impact points to pass over the pole at $\gamma = 90°$. Thus a longitude deviation which is positive for $\gamma < 90°$ is suddenly converted to a negative correction for $\gamma > 90°$. At the instant $\gamma = 90°$, what was a longitude deviation suddenly becomes a latitude deviation, as shown by the presence of an isolated point at $(0, -.81)$. To reemphasize, the behavior shown is a characteristic of the latitude-longitude coordinate system rather than the functions α_I, $\Delta\phi^{(1)}$ and τ_1.

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Three-Way "Shake Table" for Vibration Testing.—Development of a three-way "shake table" which can be used to improve reliability of missile components disclosed at The University of Michigan during a meeting of the Human Factors Society of America.

The new device also promises to be of great value in determining the physical and psychological reactions of humans in supersonic and space flight, according to its designer, T. Charles Helvey, who is a Professor of bio-physics at the University of Kansas, Lawrence, and heads the human factors research program of Radiation, Inc., in Orlando, Fla., where the "shake table" is now on the drawing boards. Helvey said the theoretical problems of designing the new device have been solved, and indicated it was only a matter of time until it reached the production stage. He estimated its cost at about $100,000.

The "shake table" will enable scientists to duplicate from 80 to 90 per cent of the vibrations which occur during missile take-offs and stretch them out over a prolonged period of time. By subjecting missile components to these vibrations for sustained periods, it will be possible to eliminate many of the break-downs which now plague missile development. The first model of the new table will be able to test parts weighing up to 1500 lb.

Human volunteers strapped to the new device can be subjected to vibrations closely resembling those of supersonic and space flight, Helvey said. This will give researchers a chance to measure physical and psychological reactions of man under these conditions.

At present, Helvey noted, all the world's "shake tables" vibrate in only one direction—up-and-down, forward-and-back, or side-to-side. The new machine will duplicate all these motions simultaneously or in any desired combinations.

Ceramic Cores for Fuel Elements.—Metal clad fuel elements containing ceramic or cermet cores (a combination of metal and ceramics) have proved extremely promising for use in nuclear reactors, according to C. E. Weber, a metallurgist at General Electric's Knolls Atomic Power Laboratory at Schenectady, N. Y. Weber said elements of this kind that were tested showed good dimensional stability and performance under irradiation.

Weber told the Geneva Conference on Peaceful Uses of Atomic Energy that such fuel elements combine high strength with high thermal conductivity and are corrosion resistant, particularly when the ceramic components are combined with stainless steel. Uranium oxide was combined with stainless steel in some of the most promising fuel elements, he reported.

To date, he said, most of the studies of non-metallic fuel elements have been made with uranium dioxide. So far, he said, this material as a fuel has shown "high temperature stability, ability to accommodate fission products with a very small expansion rate relative to alloy fuels and low mobility of the fission gas atoms."

He said that if all ceramics perform as well as uranium dioxide, these materials as a class hold great promise for future application."

POLARIZATION STUDIES WITH THORIUM OXIDE *

BY

W. E. DANFORTH [1]

ABSTRACT

Studies of electrical polarization resulting from passage of current in single crystals of thorium oxide in the 600°–1300° C. region have been extended to detailed measurements of the initial rate of rise of such polarization. An attempt is made to deduce therefrom the density and mobility of charge carriers. Existing theoretical papers on the subject are briefly discussed. Reasonable values of carrier density are obtained but mobility values appear to be too low and to vary with temperature in the wrong sense. Probe measurements show that, for values of time less than one minute, the development of polarization EMF is confined to regions near the electrodes; for long times the polarization becomes more uniformly distributed throughout the crystal. Studies with constant voltage applied to a crystal show that, above a critical value of voltage, an instability develops in which the current increases to large values.

1. INTRODUCTION

In a previous paper (1)[2] some of the main features of the polarization[3] phenomena in thorium oxide crystals were described; theoretical considerations of a general character led to the view that, while this material (in the 800°–1300° temperature range) probably conducts electricity both by electrons and ions, the ionic conduction accounts for more than 99 per cent of the charge transport.

In the present paper are recorded results of further experiments regarding the phenomenological pattern in thorium oxide; it is anticipated that the same pattern is also qualitatively relevant to other materials. Also described herein are our attempts to determine carrier density and mobility from the rate of onset of polarization.

The mathematical treatment of this subject in terms of mobile ions subject to thermal diffusion and taking account of Poisson's equation, was set forth by Jaffe (2, 3). The problem has also been attacked by Macdonald (4) and by Friauf (5) using the same initial equations. Application of the theory specifically to liquid electrolytes was done by Chang and Jaffe (6) and systematic experimental determinations of effective capacitance and resistance with liquid electrodes in an especially designed cell were carried out by Jaffe and Rider (7).

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

[3] The word *polarization* as used in this paper refers to that phenomenon in which application of a voltage to a crystal results in the formation of a counter electromotive force which persists for a measureable time after the applied voltage is removed. This kind of polarization is termed *electrolytic* since its explanation appears to involve ionic dissociation within the material. The counter EMF is caused by the translational motion of ions over large distances; this distinguishes electrolytic polarization from dielectric polarization in which the counter EMF is the *result of each atomic unit acquiring an effective dipole moment.*

Unless simplifying assumptions are made, solution of the basic differential equations describing ionic polarization presents great difficulty. All references except (2) concern themselves solely with periodic impressed voltages and simplify the problem by confining attention (both in theory and in the experiments) to the first harmonic sinusoidal solution. All references including (2) proceed upon the assumption of slowly varying carrier density, such an assumption being of course implied when flow due to thermal diffusion is said to be proportional to the first space derivative of concentration, other derivatives being neglected.

All authors agree that, for the case of small applied voltage and in the appropriate frequency range, a polarizing substance may be represented by an equivalent capacitance and resistance in series. In an unpublished work at the Bartol Research Foundation, J. F. Marshall has shown, using a Fourier integral method, that (as one might expect) the same equivalent circuit will be valid for short d-c. pulses.

2. DETERMINATION OF ION DENSITIES AND MOBILITIES FROM INITIAL SLOPE OF THE POLARIZATION CURVE

By using the theoretical expressions for the resistance and capacitance of the equivalent circuit it would appear to be possible to determine both the density of the ionic carriers and their mobility. This fortunate circumstance results from the fact that, while the resistance depends upon the product of density and mobility, *the capacitance is a function of density alone.*

The success of this determination is contingent, naturally, on the validity of a number of assumptions[4]: (1) only one species[5] of carrier is present, (2) the disturbance by the applied field of the original uniform distribution of carriers is very small, and (3) conditions at the electrodes are such that no net charge passes to or from either electrode; that is, in Friauf's terminology, both electrodes are blocked.

Under these assumptions the effective resistance and capacitance of a crystal will be given by the following expressions:

$$R = l/Anev \tag{1}$$

and

$$C = Ae\sqrt{nK}/4\sqrt{\pi kT} \quad \text{(see footnote 6)} \tag{2}$$

[4] It has been shown (5) that terms concerning formation and recombination of carriers drop out of the equations for the case here considered; *viz.*, small changes of density.

[5] Under the term *one species* we here include the case treated by Friauf (5) in which interstitials and the associated mobile vacancies are both present with different mobilities but in equal numbers. In this case the number may be determined from the capacitance but only the mean mobility is determinable.

[6] This expression is based on a model where the mobile carriers are of one sign only, the oppositely charged entities being held fixed in the lattice. If the latter are also mobile n will still refer to number of one sign only, and the expression for C will be multiplied by $\sqrt{2}$. The expression for R then becomes $l/Aen(v_1 + v_2)$ where v_1 and v_2 are the respective mobilities (5).

where

n = density of mobile carriers
v = mobility
A = area of specimen
l = length of specimen
K = dielectric constant,

the other symbols having their usual significance. If a constant voltage is applied across the equivalent circuit consisting of R and C in series the current will rise abruptly to i_0 and then decrease with time. The initial (fractional) slope (S) of this decrease will be

$$S = \frac{1}{i_0}\frac{di}{dt} = \frac{1}{RC}. \tag{3}$$

From (2) and (3) one then has as the expression for carrier density

$$n = 16\pi kT/K(RSAe)^2. \tag{4}$$

The procedure is the following. If a fixed voltage is applied to a crystal and the initial current observed, R can be determined. If, then, the initial rate (Eq. 3) of decay of current is measured, the carrier density n can be computed from Eq. 4. Having n, the mobility v can then be computed from Eq. 1. The preparation of thorium oxide specimens, their purity, etc., have been described elsewhere (1, 8). The electrodes consist of platinum films deposited by evaporation. The specimen is mounted in a vacuum furnace with a tungsten heating element (1); a tantalum enclosure about the specimen forms a near-black body for optical pyrometry.

TABLE I.

(1) T	(2) σ	(3) S	(4) n	(5) v
920	5.51	1.90	7.5×10^{16}	4.6×10^{-2}
1000	6.69	0.70	7.7×10^{17}	5.4×10^{-3}
1055	11.27	0.33	1.0×10^{19}	7.0×10^{-4}
1105	16.03	0.50	0.9×10^{19}	1.1×10^{-3}
1145	22.04	0.29	5.1×10^{19}	2.8×10^{-4}
1215	25.4	0.17	2.0×10^{19}	7.9×10^{-4}
1285	40.0×10^{-4}	0.092×10^{-4}	1.6×10^{21}	1.6×10^{-6}
° C.	$(\text{ohm-cm})^{-1}$	sec^{-1}	cm^{-3}	$\text{cm}^2\,\text{sec}^{-1}\,\text{volt}^{-1}$

The electrical measurement is of simple character. By means of a double pole mechanical contactor the sweep of an oscillograph is initiated and, a short time later, a fixed voltage is applied to the crystal. The oscillograph deflection is proportional to the current through the crystal.

The specimens were rectangular, several millimeters on a side and 1 or 2 millimeters between electrodes. The possibility that high sur-

face conductivity might introduce an error in the measurement was investigated by a guard ring arrangement; it was found that no appreciable surface current existed so that the guard ring was not necessary.

Table I shows data taken with one crystal. Columns (1) and (2) of this table give, respectively, the centigrade temperature and conductivity ($\sigma = (I/R)(l/A)$). In Column (3) are figures for the initial slope (Eq. 3); these were obtained by photographing the oscillograph trace and laying a straight edge tangent to the curve immediately after the initial rise. In Column (4) are values of carrier density computed from Eq. 4. The values of mobility given in Column 5 may then be computed by use of Eq. 1. A major point of interest in these data is the fact that the computed values of carrier density are, at least at low temperatures, of reasonable order of magnitude. Also, as one would expect, the density increases with temperature. However, a trouble is encountered here in that with further increase of temperature the values appear to be becoming unreasonably large. The density of thorium ions in the ThO_2 crystal is 2.3×10^{22}; it is therefore hardly to be entertained that at 1285° C. a carrier density of 1.6×10^{21} could exist, increasing rapidly with temperature. A further difficulty is present concerning mobility. If mobilities in ThO_2 should be of comparable order of magnitude to those of Ag^+ ions in AgCl (9), one would say that, while our mobility values at 900° are reasonable, the values in the 1200° region seem much too small. One might question, of course, why mobilities in ThO_2 should be at all comparable to those in AgCl; but even allowing this, a more serious difficulty is the decrease in temperature of the mobility values here calculated. Results similar to those in Table I have been obtained with four specimens. Fair agreement is obtained regarding conductivity and its variation with temperature; an activation energy (u in the expression $\sigma = \sigma_0 \exp(-u/kT)$ of approximately 1.1 volts is obtained. Values of initial slope S from specimen to specimen are, however, more variable. Also, in a given specimen, the slope may change considerably over periods of several days. Freshly prepared specimens, when first heated to 900° C. were found to exhibit no short time polarization; measureable values of slope (with sweep speeds as low as 1 second per inch) developed only after several hours of high temperature vacuum processing. This phenomenon may have been associated with removal of excess oxygen, or may have been due to changes in conditions at the electrode-crystal interface. This variability of initial slope, and the associated nonreproducibility of carrier density values, may be ascribed, we speculate, to uncontrolled changes of conditions of the electrodes. It would seem reasonable that, as has been expressed by Friauf (5), the flow of charge between the crystal and either electrode is neither completely free nor (as the above calculation assumes) completely blocked but that some intermediate condition exists which depends upon the precise nature

of the electrode interface. If so, one might expect the phenomena to show a high degree of variability, comparable to that of thermionic emission in a poor vacuum. In short, our initial slope measurements lead to carrier density values of the right order of magnitude but a variability of these values exists which we think is due to change with time of the precise boundary conditions which should be used in the mathematical treatment.

Further experimental effort to obtain quantitative results in this field should, it would seem, include specific attention to the definition and control of conditions at the electrode surface.

3. PROBE MEASUREMENTS OF DISTRIBUTION OF POLARIZATION POTENTIAL

The theoretical model upon which the foregoing calculations are based can be shown to lead to the result that the polarization potential difference which builds up with time is confined to narrow regions at one or both of the electrodes. It was therefore in order to determine experimentally the principal feature of the spatial distribution of polarization potential. For this purpose a cylindrical specimen was cut from a crystal of thorium oxide and lathe-ground. The cylinder was 11 mm. long and 4 mm. in diameter. The ends of the cylinder were platinum plated to provide electrodes. The probe consisted of fine platinum wire laid in grooves at a distance from each electrode equal approximately to one-fourth the length of the specimen. The grooves were platinum plated to insure good contact with the probes.

The observations were all taken under conditions of constant current. The initial stages of polarization were studied by connecting the output of the preamplifier to the input of an oscilloscope. Sweep speeds as low as 1.1 seconds per inch were used. Long-time studies, in which potential values approached equilibrium, were made by connecting the output of the preamplifier to an electronic electrometer.

All short-time studies (those in which the current flowed for less than one minute) showed, indeed, that the development of polarization potential was confined to the end regions. In Fig. 1, the straight line (1) shows the distribution of potential along the specimen at the instant a constant current of 26.8 μA was initiated. Temperature was 1070° C. A represents the grounded end of the specimen. The ordinate at B is the full potential difference across the specimen; P_A and P_B give the positions of the probes. In the case here shown, the data for curve (3) were taken 11 seconds after initiation of the current. The total potential difference across the specimen has more than doubled while that between the probes is changed little, if at all. Curve (3) is obtained by subtracting the data of curves (1) from those of curve (3) and represents therefore the "polarization EMF" which was produced by passage of current. One notices that, as well as being confined to the ends of the specimen, the polarization EMF is approximately equally distributed between the two ends.

If the current is carried by a single type of ionic carrier this symmetry would indicate that both electrodes were blocked. Symmetry would not be expected if interstitial thorium or oxygen vacancies were the carrier. Migrations of interstitial oxygen or thorium vacancies might well develop a blocked condition at the anode.

When current is allowed to pass through the crystals for those long times which are required for the EMF to approach equilibrium, the

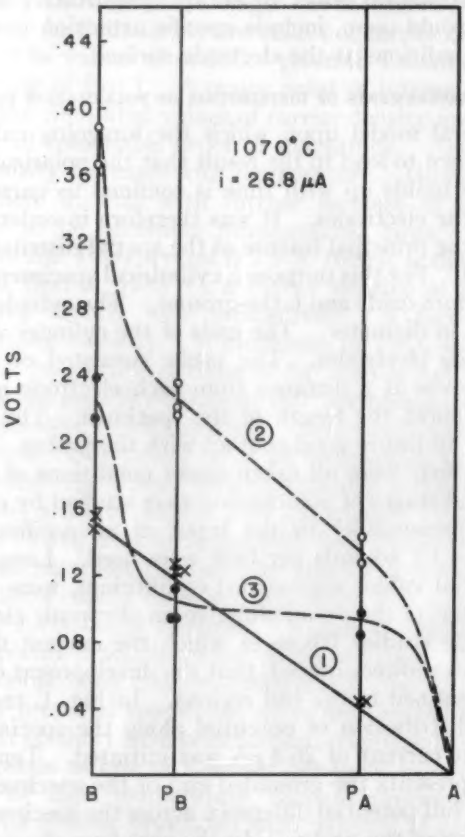

FIG. 1. Short time potential distribution.

distribution changes in a way which was unexpected and for which satisfactory explanation is still wanting.

In Fig. 2, the potentials of the probes, and of the high end of the specimen, are plotted against time in hours. In the case illustrated, 10 μA was maintained in the specimen at a temperature of 1265° C.

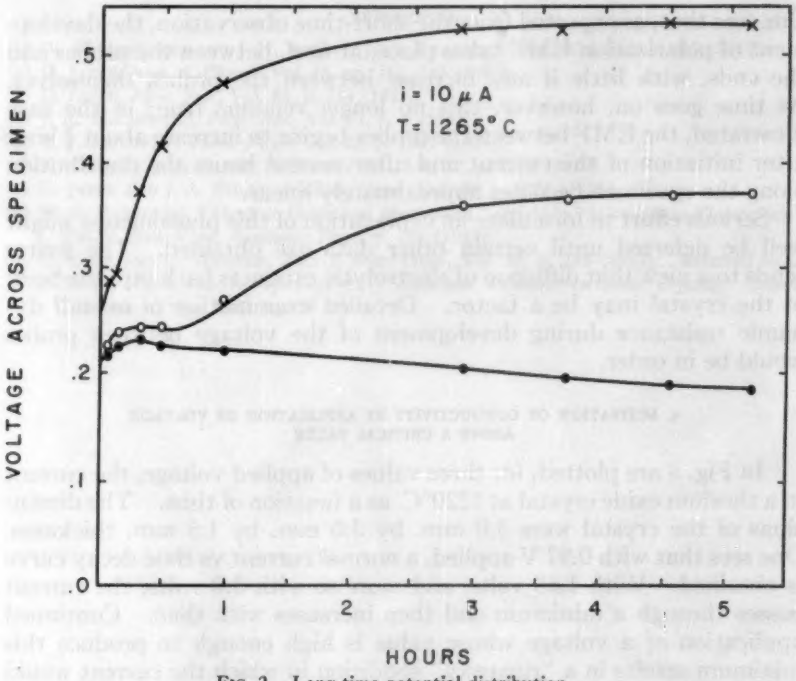

FIG. 2. Long time potential distribution.

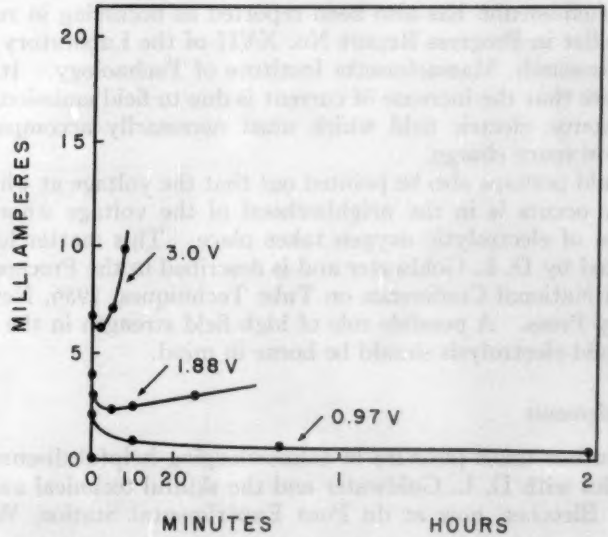

FIG. 3. Activation of conductivity when voltage exceeds a critical value.

One sees that, as expected from the short-time observation, the development of polarization EMF takes place, at first, between the probes and the ends, with little if any increase between the probes themselves. As time goes on, however, this no longer remains true; in the case illustrated, the EMF between the probes begins to increase about ¾ hour after initiation of the current and after several hours the distribution along the specimen becomes approximately linear.

Serious effort to formulate an explanation of this phenomenon might well be deferred until certain other data are obtained. The writer tends to a view that diffusion of electrolytic products back into the body of the crystal may be a factor. Detailed examination of overall dynamic resistance during development of the voltage between probes would be in order.

4. ACTIVATION OF CONDUCTIVITY BY APPLICATION OF VOLTAGE ABOVE A CRITICAL VALUE

In Fig. 3 are plotted, for three values of applied voltage, the current in a thorium oxide crystal at 1220°C. as a function of time. The dimensions of the crystal were 3.0 mm. by 3.0 mm. by 1.5 mm. thickness. One sees that with 0.97 V applied, a normal current vs time decay curve is obtained. With 1.88 volts, and more so with 3.0 volts, the current passes through a minimum and then increases with time. Continued application of a voltage whose value is high enough to produce this minimum results in a "runaway" condition in which the current would increase to very high values.

This phenomenon has also been reported as occurring in rutile by V. L. Sandler in Progress Report No. XVII of the Laboratory for Insulation Research, Massachusetts Institute of Technology. It is suggested there that the increase of current is due to field emission caused by the intense electric field which must necessarily accompany the polarization space charge.

It should perhaps also be pointed out that the voltage at which this instability occurs is in the neighborhood of the voltage where rapid production of electrolytic oxygen takes place. This matter has been investigated by D. L. Goldwater and is described in the Proceedings of the Third National Conference on Tube Techniques, 1956, New York University Press. A possible role of high field strength in the mechanism of solid electrolysis should be borne in mind.

Acknowledgments

The author takes pleasure in acknowledging helpful discussions of these topics with D. L. Goldwater and the skillful technical assistance of Harry Bleecher, now at du Pont Experimental Station, Wilmington, Del.

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Spectroheliometer.—How to make an automatic report of the sun's radiation from dawn to dusk was explained at The University of Michigan by Yro T. Sihvonen of General Motors Research Laboratories' Physics Department, at the Second National Conference on Applied Meteorology. He described the spectroheliometer, a specially designed instrument that traced a continuous record of solar power for two and a half years at Miami, Fla.

It was developed for studies of the sun's weathering effects on automotive finishes, fabrics, plastics and plating at GM Research Laboratories' Florida Test Field. Before installation at Miami, he said, it measured the sun's radiant energy over Detroit for a year so that comparative readings were obtained for both cities. The Detroit record showed sharp variations in the amount of solar radiation reaching the ground. This so-called "solar twinkle," Sihvonen stated, resulted either from foreign particles in the air or from some form of atmospheric turbulence—or both—in Detroit's industrial atmosphere. It occurred on cloudless days.

Miami's radiation pattern was much less variable because of the instrument's non-industrial surroundings.

A suitable device for long-term solar research, the spectroheliometer measures automatically not only the total amount of the sun's energy blanketing the earth but also the energy being transmitted in five different wave length bands. It is mounted like an astronomical telescope for tracking the sun from its rise until its setting. Sunlight enters it through two windowed apertures. Behind one aperture is a total radiation detector and behind the other is an optical system. The optical system separates the sunlight into its component wave lengths, and energy from each of the five different wave length bands is measured by a highly sensitive thermopile. The thermopile generates voltage proportional to the amount of heat it receives. This is fed into an amplifier, a recorder and an integrator for a continuous record of solar radiation.

Sihvonen said that in weathering studies spectroheliometer data were correlated with the fading or other deterioration that occurred to paint and fabric samples exposed to the sun. One fact uncovered by this solar energy record was that the sun's rays most destructive to paints and fabrics were the least intense energy rays. Their destructive effectiveness was high; their quantity low.

Spectroheliometer data also enabled GM researchers to develop an artificial weathering device, "Little Florida," for duplicating in the laboratory what occurs in natural atmosphere and sunlight. This accelerates the weathering process on automotive paints and finishes under controlled conditions.

From a meteorological standpoint, information from the spectroheliometer has indicated that:

1. Atmospheric absorption of ultraviolet light increases with the amount of air through which it passes. Thus, ultraviolet radiation is much stronger at noon than at sunrise or sunset.

2. At both Detroit and Miami radiation is not maximum at solar noon, as was expected, but occurs a few minutes thereafter, presumably because of the humidity variation throughout the day.

3. The value known as the solar constant—in effect, the amount of radiation striking the earth's surface—is somewhat low and should be raised. This also has been reported by U. S. Bureau of Standards scientists.

MEDAL DAY PROCEEDINGS AT THE FRANKLIN INSTITUTE

October 15, 1958

RECEPTION AND DINNER

On Wednesday, October 15, 1958, approximately 400 Institute members and guests gathered to pay tribute to the men whose achievements had been recognized by the Committee on Science and the Arts during the past year. A reception honoring the 1958 Medalists was held at 6:30 P.M. in the lobby of Franklin Hall, and at 7:10 the guests assembled for dinner in the Hall. The Reverend John C. Knewstub, Rector of Old St. David's Church, Radnor, asked the blessing.

During dinner, Mr. Wynn Laurence LePage, President of the Institute, presiding, thanked the members of the Hostess Committee for their assistance at the reception and for the floral decorations. Mr. LePage proposed the following toast to Benjamin Franklin:

"We have met here this evening for the special purpose of recognizing and honoring a selected group of men for outstanding achievement in the physical sciences.

"But in this majestic Hall, overshadowing us and seeming to lend a blessing to our worthy purpose, is the spirit of one whose character, stature and achievements have given him a place in history. His daring in investigating scientific phenomena, his wisdom in treating with his fellow men and his consciousness of the history he helped to make have set him apart. He it is who inspires this great Institute in its educational and scientific research work.

"Ladies and Gentlemen: To Benjamin Franklin!"

Mr. Joseph Gray Jackson, Chairman of the Committee on Science and the Arts, introduced the following former Medalists who were present at the dinner.

FORMER MEDALISTS ATTENDING MEDAL DAY—1958

Year	Medalist	City
	CRESSON (Founded 1848)	
1903	G. H. Clamer	Philadelphia, Pa.
1952	H. Birchard Taylor	Glen Riddle, Pa.
1957	R. J. S. Piggott	Pittsburgh, Pa.
	CERTIFICATE OF MERIT (Founded 1882)	
1921	Joseph S. Hepburn	Philadelphia, Pa.
1939	E. Burke Wilford	Philadelphia, Pa.
1942	Walter Larkin	Philadelphia, Pa.

Year	Medalist	City
	LONGSTRETH (Founded 1890)	
1902	Williams, Browne & Earle, Incorporated (Represented by Mr. Peter Abrams)	Philadelphia, Pa.
1911	Joseph S. Hepburn	Philadelphia, Pa.
1912	J. H. Granbery	Richmond, Va.
1916	The Sharples Corporation (Represented by Laurence P. Sharples)	Philadelphia, Pa.
1922	Joseph F. Keller	New York, N. Y.
1923	Harry S. Parks	Pottstown, Pa.
1937	John S. Haug	Philadelphia, Pa.
1940	Games Slayter	Newark, Ohio
1955	Richard Y. Case	Philadelphia, Pa.
1956	Floyd A. Firestone	New York, N. Y.
	POTTS (Founded 1906)	
1928	William C. Taylor	Corning, N. Y.
1948	Clarence A. Lovell	Summit, N. J.
	LEVY (Founded 1923)	
1938	Alger L. Ward	Philadelphia, Pa.
1945	Rupen Eksergian	Philadelphia, Pa.
1948	William H. Cherry	Princeton, N. J.
	HENDERSON (Founded 1924)	
1937	Rupen Eksergian	Philadelphia, Pa.
1954	C. Levon Eksergian	Philadelphia, Pa.
1955	Carleton K. Steins	Philadelphia, Pa.
	WETHERILL (Founded 1925)	
1940	Edward E. Kleinschmidt	Deerfield, Ill.
1941	Harold S. Black	Summit, N. J.
1951	Howard O. McMahon	Cambridge, Mass.
1952	Harrison P. Hood	Corning, N. Y.
1952	Albert J. Williams, Jr.	Philadelphia, Pa.
	BROWN (Founded 1938)	
1955	Charles S. Leopold	Philadelphia, Pa.
	BALLANTINE (Founded 1946)	
1952	Walter H. Brattain	Murray Hill, N. J.
1956	Kenneth Bullington	Madison, N. J.
1957	Robert M. Page	Washington, D. C.
	THE FRANKLIN MEDAL (Founded 1914)	
1942	Jerome C. Hunsaker	Cambridge, Mass.
1953	William Francis Gibbs	New York, N. Y.

MR. LEPAGE: "Thank you, Mr. Jackson, and to this distinguished group of honored Alumni of The Franklin Institute may I say how

happy we are to have you return each year and help us make this so outstanding an event in the calendar of The Institute.

"Much of the advisory work involved in running The Franklin Institute so that its great traditions may be preserved and passed on is performed by a number of committees. The oldest of these is the Committee on Science and the Arts. Mr. Jackson, from whom we have just heard, is the present Chairman of that Committee. Several years ago, it was my honor to serve in that capacity. So it is that I look forward with real pleasure to the next few moments.

"The Committee on Science and the Arts was founded during the first year of The Franklin Institute. At that time it was called the Committee on Inventions. Its purpose was to carry out one of the founding ideas of The Franklin Institute: to encourage the development of useful devices. In doing this, the Committee investigated inventions of the day and reported on them; and, by those activities, encouraged the inventors to further their work in the invention, design and development of new and useful devices and processes. In 1834, its present name was adopted and this Committee has continued to encourage men to strive for new things, new ideas and new devices that bring comfort, convenience and better living to all.

"The Committee on Science and the Arts and its various Sub-committees seek out, investigate and recommend for award men who have achieved in a superior way thus making them eligible for the awards of The Franklin Institute. The investigations of this Committee are indeed exhaustive as I, myself, can fully attest. When the members of the various Sub-committees and the main Committee itself have assured themselves that a man has indeed achieved in a superior way, he is recommended to the Board of Managers of The Franklin Institute for an award.

"Gentlemen of the Committee on Science and the Arts, would you please stand?

"Ladies and Gentlemen, will you join with me in my warm thanks to my old friends and colleagues for another year of outstanding service to The Franklin Institute."

STATED MEETING

MR. LEPAGE: "Today, October 15, 1958, marks the first Stated Monthly Meeting of The Franklin Institute of the State of Pennsylvania for this new season for all our seven thousand or more members. The minutes of the May 21, 1958 meeting were published in the June issue of the JOURNAL OF THE FRANKLIN INSTITUTE. If there are no additions or corrections to those minutes—I declare them approved as published.

"Now, it is my pleasure to present to you the Executive Vice President of The Franklin Institute, who will tell of some of the plans that

have been made and the activities that lie ahead for The Institute. Dr. John S. Burlew."

REPORT OF THE EXECUTIVE VICE PRESIDENT

DR. BURLEW: "The Franklin Institute at times seems to be a factory for producing 'events'—and so in telling you about some of the coming events, I may appear to be a salesman displaying his firm's products for the coming year. Just as the salesman looks for customer appeal, so do we at The Franklin Institute. In our case, the customers are the men, women, and children for whom the Institute attempts to furnish some new insight into the meaning of science, as one of its educational activities.

"The first event that I would like to call to your attention will take place next week. On Tuesday, October 21, The Franklin Institute will hold the third in a series of annual symposia on problems in air pollution. This one will deal with 'Odor—Measurement and Control.' One feature of the program will be the annual Cleaner Air Week luncheon sponsored by a city-wide committee.

"This symposium is one of the rare occasions when The Franklin Institute can bring to public attention the work of its Laboratories for Research and Development. In these Laboratories 'events' are taking place all through the year—events that are reported to the companies and Government agencies that support the research projects, but about which the general public is seldom informed, because of the nature of the work.

"The day after the air pollution symposium there will be another event of an entirely different sort. Professor V. E. Eaton of Wesleyan University will deliver the 33rd annual James Mapes Dodge Lecture for young people, with the subject of 'Color' as his theme. An endowed lecture such as this represents one of the ways in which a living memorial to a devoted member of the Institute can serve to augment its educational efforts. Over the years, the Dodge Lecture has been an important means by which the Institute has tried to interest young people in careers in science and engineering.

"Invitations are sent to schools throughout the metropolitan Philadelphia area to suggest qualified pupils to come to this free lecture. The response has been so great this year that we have had to ask Dr. Eaton to give two lectures the same day.

"The Winter Work Shops are another part of the Institute's work with young people. In this eighth year of these classes both for juniors (those from the age of ten through twelve) and for seniors, additional courses are being offered. The Junior Work Shop endeavors to stimulate the natural interest that children have in science by giving them an opportunity to participate personally in experiments in the physical

sciences. The seniors will be able to work with their hands in learning about radio, electronics, photography and model making. This year there is also a special course for those who wish to prepare for an amateur radio operator's license.

"Next spring we look forward to a very special event for young people—the annual Science Fair, co-sponsored by the Institute and the Philadelphia Inquirer. The Science Fair has become increasingly popular. This past spring it was necessary to have two separate Fairs—one for Juniors and one for Seniors—and we plan to repeat that pattern next April.

"At Medal Day last year I mentioned plans then underway that we had hoped would have led to an event that could have been announced this evening. Those plans were for a Basic Chemistry Exhibit in the Museum, which we urgently need to round out our presentation of the fundamentals of science to young people. We have been surprised that in this year of great public clamor for more science education we have not yet been able to raise the funds for the construction and operation of this exhibit.

"The series of evening lectures held at the Institute on most Wednesday evenings from November through May gives Philadelphians and their neighbors a chance to become acquainted with the latest developments in a wide variety of scientific fields. In the more than twenty lectures scheduled, members of the Institute and their guests can learn of the latest concepts on communications, magnetohydrodynamics, metallurgy and astronomy—to name just a few of the highlights of the coming meeting season.

"It is a particular pleasure to recall that many of these Wednesday evening meetings, which regularly include a dinner here in Franklin Hall, are held jointly with different technical societies. Of course many of the members of the Institute are also members of one or more of the technical societies; and so these joint meetings give them an opportunity to demonstrate this community of interest. This year we are pleased to add one more such joint meeting—one in March with the Philadelphia Chapter of the American Institute of Architects.

The Planetarium of The Franklin Institute continues to be a big attraction. Last year a quarter of a million persons attended the Planetarium shows and attendance this year has been running at an even greater rate. The Planetarium staff has recently started a special course called 'Astronomy for Everyone', a repetition of a very popular series of lectures and demonstrations given last year.

"There is another event for which we are planning, but for which the details cannot yet be announced. This is to be a special observance of the 25th Anniversary of the opening of The Franklin Institute Museum and Planetarium. The planetarium was opened to the public in November 1933, whereas the Museum itself was not ready until January

1, 1934. We would like you to watch for a later announcement of the celebration that is being planned for 1959."

MR. LEPAGE: "Thank you, Dr. Burlew.

"Ladies and Gentlemen: This is my first Medal Day as President of The Franklin Institute (it could well be my last!) and frankly, I have been perplexed and not a little nervous as to what I should say to such a distinguished assemblage at this, our most important meeting of the year.

"Against a backdrop of world turmoil and conflict one cannot help but be impressed by the atmosphere of mutual interest and understanding which exists among men of science the world over. Perhaps it is likewise true among men of achievement in the other arts, music, literature and the graphics. The creative instinct, when practiced, leaves no room for distrust and conflict. Is this because the creative instinct has a spiritual connotation? Probably.

"I wish that, in this great Hall, inspired by the memory of Benjamin Franklin, men of superior understanding in international affairs could meet in the same calm atmosphere that prevails when men of science from all parts of the world gather together to exchange ideas and theories concerning the origin, structure and exploration of this vast Universe. In his writings, Benjamin Franklin said that a philosopher should be able to set foot in any country of the world and call it his own.

"In the *Institute News*, I see frequent mention, in words and pictures, of scientists from countries around the world who have visited here in friendly company with our own men of science.

"In the JOURNAL, I note papers by scientists and engineers from many countries in both hemispheres, and in the listings of our past Medalists are to be found the names of nationals from everywhere.

"In the folder which you found at your place tonight, you too, will note, if only in the sound of a name the international character of our Franklin Institute family of Medalists.

"Three of our Medalists this evening are Englishmen and one an Austrian, residing in South America. In the very sound of the names of our honored Medalists you will recognize Ireland, the Ukraine, England, Scotland, Wales, France, Germany, and Scandinavia.

"The awards of The Franklin Institute are given regardless of nationality, race or creed, for superior work in the many fields of the physical sciences. Let us now proceed.

"Each 1958 award recipient is sponsored by the Chairman of the Sub-Committee of the Committee on Science and the Arts which, after long and searching study, recommended him for the award. Each Sponsor will present the recipient and read his citation. Will the Sponsors please now take thier places?"

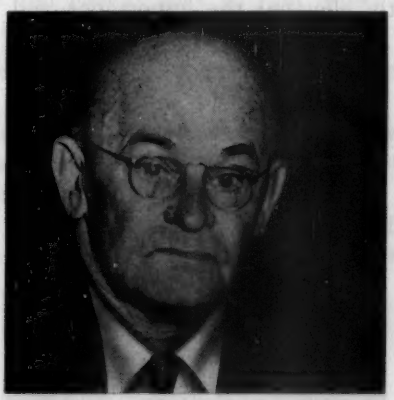
PRESENTATION OF AWARDS

PRESENTATION OF THE ELLIOTT CRESSON MEDALS

(Founded in 1848)

This Medal is awarded for discovery or original research, adding to the sum of human knowledge, irrespective of commercial value; leading and practical utilizations of discovery; and invention, methods or products embodying substantial elements of leadership in their respective classes, or unusual skill or perfection in workmanship.

To Joseph C. Patrick, Consultant, Thiokol Chemical Corporation, Trenton, New Jersey; and to S. Timoshenko, Professor Emeritus, Stanford University, Stanford, California.



JOSEPH C. PATRICK

MR LEPAGE: "The Chair recognizes Dr. Joseph S. Hepburn."

DR. HEPBURN: "Mr. President, I present Joseph C. Patrick for an award.

"In 1924 Dr. Patrick began his extensive researches on the combination of inorganic sulfides with certain organic chemical compounds containing the element chlorine. He obtained as a product a synthetic polysulfide polymer having the properties of rubber. He devised new processes to carry out these reactions on a commercial scale, and thus made possible the manufacture of a synthetic rubber having special properties adapted to critical uses and to the manufacture of a solid rocket fuel.

"I present Joseph C. Patrick, of Yardley, Pennsylvania, as a candidate for an Elliott Cresson Medal, 'In consideration of his discoveries in the field of polysulfide polymers, and of the new processes devised by him involving the combination of chemical compounds for the

manufacture of a synthetic rubber having special properties adapted to critical uses and to the manufacture of a solid rocket fuel.' "

MR. LEPAGE: "Thank you, Dr. Hepburn.

"On behalf of The Franklin Institute of the State of Pennsylvania, I present to you, Dr. Patrick, an Elliott Cresson Medal and the Certificate and Report which accompany it.

"The Chair recognizes Dr. Rupen Eksergian."

DR. EKSERGIAN: "Mr. President, I present, *in absentia*, Stephen P. Timoshenko for an award.

"Not only in this country but abroad, Professor Timoshenko is well known as an outstanding authority in the field of applied mechanics. His influence, in our university curriculum, both as an author and teacher, has greatly elevated the importance and status of applied

STEPHEN P. TIMOSHENKO

mechanics as a basic subject in our present engineering education. His contributions as an author (and his promotion of applied mechanics through our engineering societies) are well recognized by the entire engineering profession.

"His direct scientific contributions, through a group of important papers of permanent value have advanced our basic knowledge in the fields of elasticity and elastic stability. An important phase in his many contributions to engineering science is the introduction of advance generalized methods used in mathematics and physics but ingeniously transformed to a useful formulation for analytical design, particularly adaptable to the application and solution of technical problems in the theory of structures, strength of materials, vibration analysis and machine design.

"In his honor Stanford University has established The Timoshenko

Laboratory of Engineering Mechanics for research in fields of applied mechanics. He has been honored as the first recipient of the Timoshenko Medal, recently established through the Applied Mechanics Division of the American Society of Mechanical Engineers.

"I present Stephen P. Timoshenko, of Palo Alto, California, as a candidate for an Elliott Cresson Medal, 'In consideration of his contribution to Applied Mechanics as an author and teacher, and for his original work in the Theory of Elasticity and Elastic Stability.' "

MR. LEPAGE: "Thank you, Dr. Eksergian.

"On behalf of The Franklin Institute of the State of Pennsylvania, I present to you, *in absentia*, Dr. Timoshenko, an Elliott Cresson Medal and the Certificate and Report which accompany it. In the absence of Dr. Timoshenko, will you, Gregory Timoshenko, carry this Cresson Medal, Certificate and Report back to your father and present them to him for us? I thank you."

PRESENTATION OF THE CERTIFICATE OF MERIT

(Founded in 1882)

This Certificate is awarded to persons adjudged worthy thereof for meritorious inventions, discoveries or improvements in physical processes or devices.

To Harold Lyons, Head, Atomic Physics Department, Hughes Aircraft Company, Culver City, California.

HAROLD LYONS

MR. LEPAGE: "The Chair recognizes Mr. John H. Neher."

MR. NEHER: "Mr. President, I present Harold Lyons for an award.

"The advance of science and technology has always been accompanied by the requirement for ever increasing precision or accuracy in measurement of the basic units involved. Unfortunately our present

time and frequency standards leave much to be desired in the way of absolute accuracy and invariant reproducibility.

"Heretofore these standards have been based on astronomical determinations of the period of rotation of the Earth. It is known, however, that the Earth is gradually slowing down, and we now find it necessary to develop a new standard, by which we can check the variations in our previous standard, the period of rotation of the Earth.

"In recent years, vibrations of atoms in molecules—or what are more specifically termed spectral lines—have been found with microwave region, and it has been possible to make very precise measurements of these lines using electronic equipment of unprecedented sensitivity. This technique furnishes the basis for the atomic clock which has now been developed to a point where the error is less than one second in three hundred years.

"Dr. Lyons was instrumental in constructing the first of these atomic clocks in 1948, and he has actively engaged in their further development.

"I present Harold Lyons, of Culver City, California, as a candidate for a Certificate of Merit, 'In consideration of his work in pioneering the development of clocks of very great precision based on the natural periods of vibration of atoms and employing microwave techniques.' "

MR. LEPAGE: "Thank you Mr. Neher.

"On behalf of The Franklin Institute of the State of Pennsylvania, I present to you, Dr. Lyons, a Certificate of Merit and the Report which accompanies it."

PRESENTATION OF THE EDWARD LONGSTRETH MEDALS
(Founded in 1890)

This Medal is awarded for inventions of high order and for particularly meritorious improvements and developments in machines and mechanical processes.

To James Bailey, Consultant, Plax Corporation, Hartford, Connecticut; to George S. Crampton, Ophthalmologist, Philadelphia, Pennsylvania; and to Price C. McLemore, Planter, Agricultural Consultant and Inventor, Montgomery, Alabama.

MR. LEPAGE: "The Chair recognizes Dr. Julian W. Hill."

DR. HILL: "Mr. President, I present James Bailey for an award.

"The achievements that occasion this award have to do with techniques and machines for converting plastics into useful shapes. As Director of Research of Plax Corporation, Mr. Bailey led a group that in the period 1937 to about 1950 made a variety of significant advances in the processing of plastics. Of these advances, the most far-reaching and conspicuous was the commercially successful manu-

facture of plastic bottles. Their lightness and unbreakability and, when made from flexible plastics, their squeezability have led to their widespread use as convenient containers for such things as medicines, cosmetics, and insecticides, and for the safe storage and transport of dangerous liquids.

"Another advance contributed by Mr. Bailey was in the preparation of thin sheeting of normally brittle plastics, for example, polystyrene, by extruding and stretching the sheeting in such a way that it becomes sufficiently tough and pliable to be useful. Among other advances was a technique of extruding plastics in thick sections in accurate dimensions. Mr. Bailey's direct contributions are embodied in twenty-seven patents that have been issued to him.

JAMES BAILEY

"I present James Bailey, of Hartford, Connecticut, as a candidate for an Edward Longstreth Medal, 'In consideration of the many advances in the art of processing plastics based on his personal technical contributions and his effective leadership in research and development, and particularly of the development under his direction of methods for the successful commercial manufacture of unbreakable plastic bottles.' "

MR. LEPAGE: "Thank you, Dr. Hill.

"On behalf of The Franklin Institute of the State of Pennsylvania, I present to you, Mr. Bailey, this Edward Longstreth Medal and the Certificate and Report which accompany it."

MR. LEPAGE: "The Chair recognizes Mr. Lionel F. Levy."[1]

MR. LEVY: "Mr. President, I present George S. Crampton for an award.

[1] Mr. Lionel F. Levy acted as Sponsor in place of Dr. John Frazer, Chairman of the Sub-Committee.

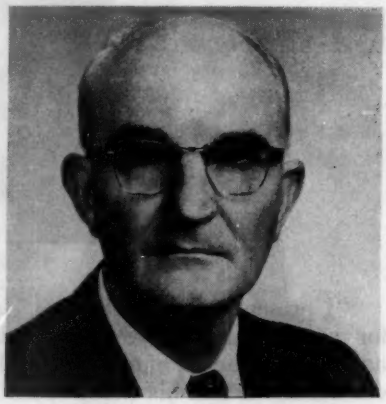
"Eminent ophthalmologist, indefatigable worker and creator of the 'Borescope,' an ingenious and necessary instrument permitting of the internal inspection and detection of flaws in normally closed vessels, he lent a helping hand to the Illuminating Engineering Society, of which he was National President in 1920–1921, and by which Society, for meritorious contributions, he was awarded its Medal in 1956.

"I present George S. Crampton, of Philadelphia, Pennsylvania, as a candidate for an Edward Longstreth Medal, 'In consideration of his development of the "Borescope," a specialized periscope for the internal inspection of turbine shafts, rotors, guns, and various closed vessels, by which flaws may be detected thus providing protection of vital importance from the physical and economic hazards of operational failure.' "

MR. LEPAGE: "Thank you, Mr. Levy.

"On behalf of The Franklin Institute of the State of Pennsylvania I present to you Dr. Crampton, this Edward Longstreth Medal and the Certificate and Report which accompany it.' "

GEORGE S. CRAMPTON



PRICE C. MCLEMORE

MR. LEPAGE: "The Chair recognizes Dr. William G. Schmidt."[2]

DR. SCHMIDT: "Mr. President, I present Price C. McLemore for an award.

"Colonel McLemore has pioneered a further important step in mechanizing the cultivation of cotton, which began one hundred and seventy years ago with the invention of Eli Whitney's cotton gin.

"I present Price C. McLemore, of Montgomery, Alabama, as a candidate for an Edward Longstreth Medal, 'For his successful application of selective flame cultivation to reduce the life expectancy of

[2] Dr. William G. Schmidt acted as Sponsor in place of Mr. Joseph Gray Jackson, Chairman of the Sub-Committee.

noxious growths in a cotton field together with the large reduction in labor costs, and the fostering of more complete farm mechanization.' "

MR LEPAGE: "Thank you, Dr. Schmidt.

"On behalf of The Franklin Institute of the State of Pennsylvania, I present to you, Mr. McLemore, this Edward Longstreth Medal and the Certificate and Report which accompany it."

PRESENTATION OF THE HOWARD N. POTTS MEDALS
(Founded in 1906)

This Medal is awarded for distinguished work in science or the arts; important development of previous basic discoveries, inventions or products of superior excellence or utilizing important principles.

To W. Nelson Goodwin, Jr., Consultant, Weston Electrical Instrument Corp., Newark, New Jersey; and to Emanuel Rosenberg, Electrical Engineer, Bogotá, Colombia, South America.

MR. LEPAGE: "The Chair recognizes Dr. Raymond C. Machler."[1]

DR. MACHLER: "Mr. President, I present William Nelson Goodwin, Jr., for an award.

"Mr. Goodwin has been associated with the design and testing of electrical measuring instruments for the past sixty years. He became employed with the Weston Electrical Instrument Corporation in 1898, first in the Engineering Department, then as Chief Engineer, and finally as Vice President and Director of Research. Although retired, he is still connected with the company as consultant.

"During this period Mr. Goodwin invented and perfected the thermal ammeter, an instrument that uses the heat generated by the current to indicate its quantity. It is particularly useful for measuring currents of high frequency. The rectifier instrument is also an invention of his, the rectifier enabling the application of a direct current instrument with its inherent higher sensitivity, to alternating current measurement. He invented and perfected a vacuum tube tester from which the desired tube characteristic could be read directly from a scale, the mathematical computation being an inherent feature of the design. And finally, he invented and perfected the photographic exposure meter.

"I present William Nelson Goodwin, Jr., of New Hope, Pennsylvania, as a candidate for a Howard N. Potts Medal, 'In consideration of his many contributions to the field of electrical measuring instruments of which the thermal ammeter, the vacuum tube tester, and the rectifier instrument are examples, and in particular for his work on photoelectric exposure meters and establishing a scale for film speeds.' "

[1] Dr. Raymond C. Machler acted as Sponsor in place of Dr. Benjamin J. Wilson, Chairman of the Sub-Committee.

MR. LEPAGE: "Thank you, Dr. Machler.

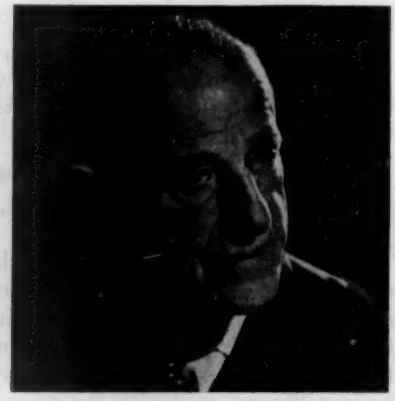
"On behalf of The Franklin Institute of the State of Pennsylvania, I present to you Mr. Goodwin, this Howard N. Potts Medal and the Certificate and Report which accompany it."

MR. LEPAGE: "The Chair recognizes Mr. J. Philip Evans."

MR. EVANS: "Mr. President, I present Emanuel Rosenberg for an award.

"Dr. Rosenberg is a distinguished inventor, engineer, teacher and author of many technical reports and books, and resides in Bogotá, Colombia.

W. NELSON GOODWIN, JR.



EMANUEL ROSENBERG

"During the past fifty-four years, in various capacities, he has contributed immeasurably to the fund of scientific knowledge in the field of electrical engineering. He has been the recipient of many outstanding awards, the most recent having been given to him by the Technical University of Vienna, Austria in which he was awarded the title 'Doctor Honorable of Technical Sciences.'

"His invention of the Cross-Field Dynamo was the result of painstaking efforts to develop a new type of generator which would produce a constant output with a greatly varying velocity and which could be self excited.

"This invention made possible for the first time the electric lighting of railway passenger cars and the principle involved in the Dynamo paved the way for future developments in producing electrical machines used for electric welding, and arc lighting.

"During recent years, the Rosenberg principle has made possible the development of the Metadyne and the Ampledyne which have found wide usefulness in modern servomechanisms in the national guided missile program. There are numerous other applications of

this principle which have occurred since Dr. Rosenberg's invention, which have attested to its lasting utility.

"I present Emanuel Rosenberg, of Bogotá, Colombia, as a candidate for a Howard N. Potts Medal, 'For his conception of a fundamentally new principle for deriving electrical current from a rotating machine and for his solution of the many technical problems related to the development of the Cross-Field Generator.' "

MR. LEPAGE: "Thank you, Mr. Evans.

"On behalf of The Franklin Institute of the State of Pennsylvania, I present to you, Dr. Rosenberg, this Howard N. Potts Medal and the Certificate and Report which accompany it."

ARTHUR H. MOREY

PRESENTATION OF THE GEORGE R. HENDERSON MEDAL
(Founded in 1924)

This Medal is awarded for meritorious inventions or discoveries in the field of Railway Engineering.

To Arthur H. Morey, Manager, Locomotive Advance Engineering, Locomotive and Car Equipment Department, General Electric Company, Erie, Pennsylvania.

MR. LEPAGE: "The Chair recognizes Mr. R. Thomas Sawyer." [4]

MR. SAWYER: "Mr. President, I present Arthur H. Morey for an award.

"Mr. Morey, through widely diversified experience in his specialty of locomotive power plant engineering, covering such phases as design of steam locomotives, gas turbine power plant development, steam turbine electric locomotive, design of locomotive components on gas

[4] Mr. R. Thomas Sawyer acted as Sponsor in place of Dr. Rupen Eksergian, Chairman of the Standing Sub-Committee.

turbine electric locomotives, diesel electric locomotives and electric locomotives—some of which presented many new and unusual problems in equipment design, mechanical design of cabs and running gear and coordination of equipment design and mechanical design—direction and coordination of building and testing locomotives, has produced many outstanding accomplishments, such as the gas turbine locomotives now in use by the Union Pacific Railroad. These are but a few of Mr. Morey's achievements in the field of railroad motive power development.

"I present Arthur H. Morey, of Erie, Pennsylvania, as a candidate for the George R. Henderson Medal, 'In consideration of his engineering contributions to, and his effective leadership in, the overall design, development and service testing of the successful 4500 h.p. General Electric Company Gas Turbine-Electric Locomotives which involved the close and skillful coordination of the specialized developments by the Gas Turbine, and other component departments of the General Electric Company.' "

MR. LEPAGE: "Thank you, Mr. Sawyer.

"On behalf of The Franklin Institute of the State of Pennsylvania, I present to you, Mr. Morey, this George R. Henderson Medal and the Certificate and Report which accompany it."

PRESENTATION OF THE JOHN PRICE WETHERILL MEDALS
(Founded in 1925)

This Medal is awarded for discovery or invention in the physical sciences or for new and important combinations of principles or methods already known.

To H. A. H. Boot, Senior Principal Scientific Officer, Services Electronics Research Laboratory, Baldock, England
J. T. Randall, Wheatstone Professor of Physics, University of London, King's College, London, England; and
J. Sayers, Professor, Electron Physics Department, The University of Birmingham, Birmingham, England.

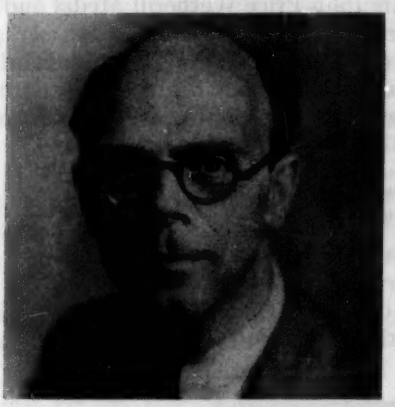
MR. LEPAGE: "The Chair recognizes Mr. Alan C. Byers."[6]

MR. BYERS: "Mr. President, I present J. T. Randall, J. Sayers, and, *in absentia*, H. A. H. Boot for awards.

"From its first employment by Great Britain in the Battle of Britain radar frequently provided to the Allied Powers the margin between victory and defeat. That the Allied Powers achieved and maintained a superior position in their radar developments was ascribable to the tireless devotion, keen imagination, and willing cooperation of numerous scientific workers in our wartime laboratories.

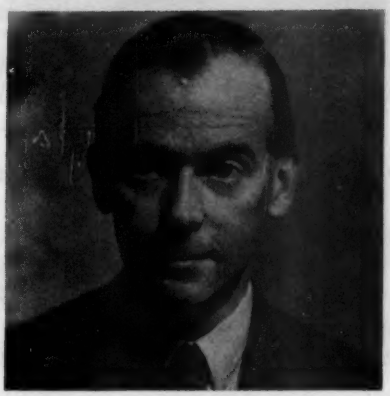
[6] Mr. Alan C. Byers acted as Sponsor in place of Mr. J. Philip Evans, Chairman of the Sub-Committee.

"The development of the high-power multicavity magnetron, for use at microwave frequencies was, if not preeminent, certainly one of the outstandingly important developments of the war years. Since it provided a high-power transmitter on frequencies much higher than those previously used, it permitted greatly improved resolution, better low-level coverage, greater accuracy, and lessened vulnerability to



J. T. RANDALL

J. SAYERS



H. A. H. BOOT

jamming. Very nearly all radar systems since that development have used magnetrons based upon the fundamental principles which were incorporated in the earliest British tubes developed by Randall, Boot and Sayers.

"I present J. T. Randall, of London; J. Sayers, of Birmingham; and, *in absentia*, H. A. H. Boot, of Baldock, England, 'For their basic developments which provided the first useful highpower pulsed micro-

wave magnetron, and which established the fundamental principles upon which all later developments in this field were based.' "

MR. LEPAGE: "Thank you, Mr. Byers.

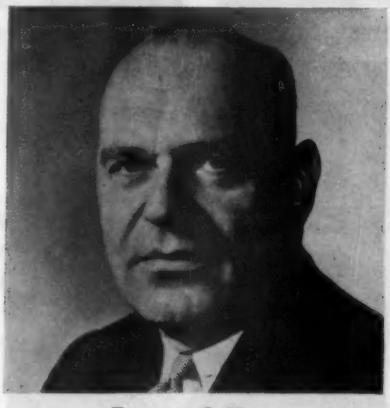
"On behalf of The Franklin Institute of the State of Pennsylvania, I present to you, Dr. Randall, this John Price Wetherill Medal and the Certificate and Report which accompany it.

"Dr. Sayers, to you, I present this John Price Wetherill Medal and the Certificate and Report which accompany it.

"And Gentlemen, in the absence of Dr. Boot, will you, for The Franklin Institute, carry his Wetherill Medal, Certificate and Report back to England with you and present them to him for us? We shall be deeply grateful to you."

PRESENTATION OF THE WALTON CLARK MEDAL
(Founded in 1926)

This Medal is awarded to the "author of the most notable advance in knowledge or improvement in apparatus, or in method concerning the science or the art of gas manufacture or distribution or utilization in the production of illumination, or of heat, or of power."



FREDERIC O. HESS

To Frederic O. Hess, President, Selas Corporation of America, Dresher, Pennsylvania.

MR. LEPAGE: "The Chair recognizes Mr. Edward G. Boyer."

MR. BOYER: "Mr. President, I present Frederic O. Hess for an award.

"Our candidate has devoted almost his entire business life to the use of gas industrially. Early in his career he developed a program, the logic of which was simple. In effect it stated

"1. Gas, an inherently controllable fuel, is subject to closer control than is being utilized, and its control can be applied over a wider range of combustion temperatures.

"2. This means that new opportunities can be found for gas application.

"The course of action was, therefore, a series of studies which would lead to the development of improved combustion equipment. This philosophy came to be an established approach in gas engineering. In time, sufficient new combustion tools (burners) had been developed to give entrée in a practical sense to an industry using heat processing; sufficient performances were on record to carry convictions in decisions of heat-users to order similar installations.

"Performances then had a scope, with respect to a variety of problems, wide enough that industries continued to present problems for which no parallel solutions had been found.

"Much of his work was solving these problems.

"Mr. Hess has had fifty-eight patents issued in his name, and an additional twenty-five patents as a joint inventor. He is the author of some fifty-five papers on industrial uses of gas.

"I present Frederic O. Hess, of Dresher, Pennsylvania, as a candidate for the Walton Clark Medal, 'In consideration of his many contributions to the gas industry, in the improvements and developments of new processes for the use of gas industrially.' "

MR. LEPAGE: "Thank you, Mr. Boyer.

"On behalf of The Franklin Institute of the State of Pennsylvania, I present to you, Mr. Hess, this Walton Clark Medal and the Certificate and Report which accompany it."

PRESENTATION OF THE FRANK P. BROWN MEDAL
(Founded in 1938)

This Medal is awarded to inventors for discoveries and inventions involving meritorious improvements in the building and allied industries.

To Charles M. Spofford, Professor Emeritus, Massachusetts Institute of Technology, Cambridge, Massachusetts.

MR. LEPAGE: "The Chair recognizes Mr. Herbert H. Swinburne."[6]

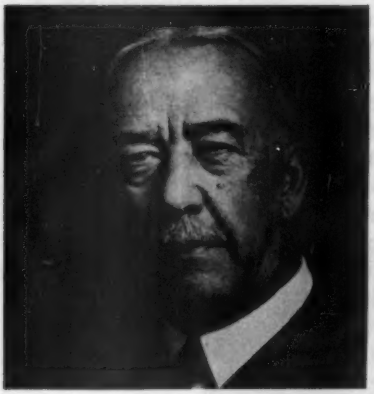
MR. SWINBURNE: "Mr. President, I present, *in absentia*, Charles Milton Spofford, of Brookline, Massachusetts, as a candidate for the Frank P. Brown Medal, 'For the engineering, aesthetic and educational accomplishments of his life work, exemplified in his prompt recognition, epousal and clarification of valid structural theories; his design of many

[6] Mr. Herbert H. Swinburne acted as Sponsor in place of Mr. Charles H. Topping, Chairman of the Standing Sub-Committee.

large and beautiful bridges, and other prominent engineering works; his contributions to engineering education through teaching and administration of an engineering educational department; and his authorship of widely used textbooks!' "

MR. LEPAGE: "Thank you, Mr. Swinburne.

"In the absence of Professor Spofford, I call upon his daughter, Mrs. Walter J. Beadle, to receive it for him. Mrs. Beadle, on behalf of The Franklin Institute of the State of Pennsylvania, I present to Charles Milton Spofford this Frank P. Brown Medal and the Certificate and Report which accompany it. Mrs. Beadle, I thank you for being with us and send with you our warmest greetings to your father."



CHARLES M. SPOFFORD

JULIAN MILES AVERY

PRESENTATION OF THE FRANCIS J. CLAMER MEDAL
(Founded in 1943)

This Medal is awarded at least once in five years for meritorious achievement in the field of Metallurgy.

To Julian Miles Avery, Metallurgican and Chemical Consultant, Ethyl Corporation, New York, New York.

MR. LEPAGE: "The Chair recognizes Mr. Francis B. Foley."

MR. FOLEY: "Mr. President, I present Julian Miles Avery for an award.

"In 1957 this country produced over 112 million tons of steel. This steel is, generally speaking, over 95 per cent iron. The source of this iron is iron ore from which the iron destined for steelmaking is reduced in blast furnaces. Starting with an idea, patented by him in 1936, Julian Miles Avery, by persistent effort, was able to demonstrate to the steel industry during World War II that a 15 per cent increase in blast furnace production of pig-iron could be accomplished with a

decrease in the cost per ton by throttling the effluent gases and thus, with comparatively little expenditure, increase the pressure within the stack. Since that time practically every new furnace built in this country or abroad has been provided with facilities for operation under the high top pressure principles discovered by Mr. Avery. Most of the pig-iron produced in the U.S.S.R. now comes from furnaces operating with high top pressure.

"I present Julian Miles Avery, of Greenwich, Connecticut, as a candidate for the Francis J. Clamer Medal, 'In consideration of his discovery and application of the principles of high top-pressure operation of blast furnaces which have very materially increased their production and contributed to their smoother and more economical operation.'"

MR. LEPAGE: "Thank you, Mr. Foley.

"On behalf of The Franklin Institute of the State of Pennsylvania, I present to you, Mr. Avery, this Francis J. Clamer Medal and the Certificate and Report which accompany it."

PRESENTATION OF THE STUART BALLANTINE MEDAL
(Founded in 1946)

This Medal is to be awarded in recognition of outstanding achievement in the fields of Communication and Reconnaissance which employ electromagnetic radiation.

To Harald T. Friis, Consulting Engineer, Rumson, New Jersey.

HARALD T. FRIIS

MR. LEPAGE: "The Chair recognizes Mr. Edward L. Forstall."

MR. FORSTALL: "Mr. President, I present Harald Trap Friis for an award.

"When our Medalist chose radio communications for his life work he was in the right place at the right time. For this field was on the threshold of tremendous expansion because of that magic 'bottle'—the electron tube. However, such expansion would not have occurred without the perseverance, the thoroughness, the genius of men such as he.

"His improvements in radio receivers contributed much to the successful inauguration and growth of transoceanic radiotelephony. But more than that, such ultra-sensitive receivers were able to detect radiations from outer space thus founding that new and surprising science—radio astronomy.

"His work on microwaves has had the most far-reaching effects on long distance communications today. Through his individual work on antennas and his overall direction of the development of the complete systems, this country is now criss-crossed by a microwave network. This vast network provides on each of its many routes for the simultaneous transmission of thousands of telephone conversations, together with television and radio programs. He was a major contributor to this revolution in long distance communications.

"I present Harald Trap Friis, of Rumson, New Jersey, as a candidate for the Ballantine Medal, 'In consideration of his many important contributions to the science of radio-communications during a lifetime of consistently productive research in this field, through which this science has been so notably advanced.' "

MR. LEPAGE: "Thank you, Mr. Forstall.

"On behalf of The Franklin Institute of the State of Pennsylvania, I present to you, Dr. Friis, this Stuart Ballantine Medal and the Certificate and Report which accompany it."

PRESENTATION OF THE FRANKLIN MEDAL
(Founded in 1914)

This Medal is awarded annually to those workers in physical science or technology, without regard to country, whose efforts, in the opinion of the Institute, acting through its Committee on Science and the Arts, have done most to advance a knowledge of physical science or its applications.

To Donald Wills Douglas, Chairman of the Board, Douglas Aircraft Company, Inc., Santa Monica, California.

MR. LEPAGE: "The Chair recognizes Dr. Harvey A. Neville."

DR. NEVILLE: "Mr. President, I have the honor to present to you as a candidate for the highest award of The Franklin Institute, the distinguished aircraft designer and aeronautical engineer, Donald Wills Douglas.

"Mr. Douglas has been a pioneer and leader in the aircraft industry

since the day he observed the Wright Brothers demonstrate their machine to the Army at Fort Myer more than a half century ago. In research and creative design he has greatly influenced the trend of aviation development throughout the world. His 'DC' series of transport planes have standardized a configuration that created an era in air travel comparable with competitive rail and bus systems.

"The tremendous popularity of the Douglas DC transport aircraft is founded squarely upon the exceptional characteristics incorporated in the entire design. These characteristics of high performance, ruggedness and economy have particularly epitomized the DC-3—a plane whose record has no parallel. One DC-3 is known to have flown over 50,000 hours and traveled seven and one half million miles; it is still in operation.

DONALD WILLS DOUGLAS

"In the field of military aviation, the genius of Mr. Douglas has likewise led the way both in military transport and combat aircraft. The C-124 Globemaster is the world's largest passenger air carrier and the Douglas Thor is the first production 1500-mile range ballistic missile.

"The eminence of Donald Douglas as a creative engineer and as the head of a major aircraft establishment has been widely recognized and suitably rewarded in this country and abroad. He has held many high offices in the industrial, civic and national associations dedicated to the promotion of aviation.

"Therefore, Mr. President, with this formal citation: 'In recognition of his creative engineering in the field of aeronautical design as exemplified in all of his work but particularly in the DC series of transport airplanes which have become standard equipment on airlines throughout

the world and have set the pattern of design in this field; and in recognition of his engineering genius which has provided inspired leadership for the technical developments of his industrial organization,' I present Donald Wills Douglas, of Santa Monica, California, as the candidate for The Franklin Medal."

MR. LEPAGE: "Thank you, Dr. Neville.

"Mr. Douglas, as President of The Franklin Institute of the State of Pennsylvania, I am greatly honored, and it gives me extreme pleasure to present to you this highest award of The Institute, The Franklin Medal and the Certificate and Report which accompany it. At its last Stated Meeting, the Board of Managers, voting unanimously, designated you, Sir, as an honorary member of The Franklin Institute. I tender you herewith your membership card and Certificate of Membership. My most sincere congratulations, Sir."

* * *

MR. LEPAGE: "Ladies and Gentlemen, in the judgment of The Franklin Institute, through its Board of Managers and on recommendation of its Committee on Science and the Arts, these are the men, present here and *in absentia*, who have achieved greatly, who have contributed much to the comfort, safety and well being of all men, who have shared the fruits of their labors and investigations, disregarding national boundaries and who have met with their fellows in all lands in the calm atmosphere in which the minds of scientists are most productive. They are indeed welcome always at The Franklin Institute. Now, Ladies and Gentlemen, as an aeronautical engineer myself, I take great pride in introducing our speaker of the evening, aeronautical engineer, pioneer in aircraft design, and the Chairman of the Board of the Douglas Aircraft Company of Santa Monica, California—Franklin Medalist, Mr. Donald Wills Douglas."

ADDRESS BY DONALD WILLS DOUGLAS

(See page 431 of this JOURNAL for the text of Mr. Douglas' address.)

After Mr. Douglas had delivered his talk, Mr. LePage conveyed to him the thanks of the Institute, dismissed the assembly with a cordial word and declared the meeting adjourned.

THE FRANKLIN INSTITUTE

MINUTES OF THE STATED MEETING

November 19, 1958

The Stated Meeting of The Franklin Institute was held at 8:15 P.M. in the Lecture Hall. Mr. Wynn Laurence LePage, President, called the meeting to order with approximately 315 members and guests in attendance.

The President stated that the minutes of the Stated Meeting of October 15, 1958 had been printed in the November issue of the JOURNAL OF THE FRANKLIN INSTITUTE. There being no corrections or additions, the minutes were approved as published.

The President recognized Mr. Morton Gibbons-Neff, Chairman of the Membership Committee, who introduced Mr. William M. C. Kimber, the one member of The Franklin Institute who had completed fifty years of membership in 1958. The President presented a gold key and gold membership card to Mr. Kimber whom we were honored to have with us.

The President then introduced the speaker of the evening, Mr. Hilliard W. Paige, General Manager, Missile and Space Vehicle Department, General Electric Company, for this, the third Charles S. Redding Lecture. The speaker's subject, he noted, was a most fascinating one because it depicts so fully the time in which we live. He then introduced Mr. Paige, who spoke on "The Technology of Manned Return from Outer Space."

Mr. Paige's talk concerning the challenge of carrying a human being across the space frontier to circle the earth and ultimately return him safely to the surface, brought out the scientific and engineering problems involved in developing this type of vehicle and the study of the type of re-entry, location and recovery in manned space travel. A large model of a nose cone developed by his company, on display in the Lecture Hall, was explained within the limits of security, by Mr. Paige. The talk was exceedingly well received as evidenced by the many questions asked during the discussion period which followed the lecture.

The meeting was adjourned by the President at 9:30 P.M.

WILLIAM F. JACKSON, JR.
Secretary

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the Library. Duplicates received will be transferred to other libraries as gifts of the donor. 353 items have been added the past month.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open from 9 A.M. until 5 P.M. on Mondays, Tuesdays, Thursdays, Fridays and Saturdays; and 2 P.M. until 10 P.M. on Wednesdays.

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DO YOU KNOW OF THE FRANKLIN INSTITUTE'S NEED FOR FINANCIAL SUPPORT

Since 1824 The Franklin Institute, through wise and conservative management, has been partially self-supporting.

As an old institution, established in the public interest, The Franklin Institute has tried to earn most of its cost by diligent and productive work. Like many other institutions of a non-profit nature, The Institute has for many years found it increasingly difficult to perform the broad educational services of which the growing nation and territories are ever in need

The educational service rendered by The Institute begins with the early grades of our elementary schools and continues throughout an individual's professional and industrial career. This period represents a span of fifty years.

Despite an erroneous public impression gained, perhaps, by the excellence and wide range of services performed, that it is a wealthy, richly endowed organization, The Institute always needs additional funds to help it perform its broad educational program.

There is a warm satisfaction in giving financial support to an organization that will use it constructively to help advance and improve our type of civilization.

Your gift or bequest, large or small, will be deeply appreciated. We shall be pleased to give any additional information regarding gifts or memorials.

Where property is transferred, title should be in the name of The Franklin Institute of the State of Pennsylvania for the Promotion of the Mechanic Arts.

Will you please write to: The Secretary, The Franklin Institute, Benjamin Franklin Parkway at Twentieth Street, Philadelphia 3, Pennsylvania, for any additional information.

MEMBERSHIP

The following elections were approved on October 2, 1958 by the Membership Committee by authority given to it by the Board of Managers.

ANNUAL

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Gilbert J. Bates, Sr.
Oswald R. Bergmann
Donald Bowerman
Miriam Braile
William W. Brown, Jr.
G. Richard Brownlee
Alexander M. Burke
Ralph J. Cheney
William D. Clements, Jr.
Luther C. Conant, Jr.
Harry S. Cook
James M. Copeland
William J. Didycz
James W. Dougherty
Edward J. Driver, Jr.
Gerhart C. Duerr
James T. Ferri
Samuel R. Frankel
Charles T. Gamble
Mrs. J. Holcombe Genung
Mrs. Percival H. Granger
DeWayne K. Guhn
Robert B. Haig
Cecil Harris
H. Frank Hentschell
Mrs. T. E. Hessenbruch
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Ralph Kraemer
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John F. McCoy
Charles F. Mengers
Baldwin Prickett
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Alfred A. Schmidt
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Sinchi Tonooka
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Alfred G. Webb
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Andrew M. Nemeth
John J. Schneider
S. Theodore Woal

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Howard G. Evans

NECROLOGY

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Joseph Berliner '54
James M. Castle '47
Albert K. A. Christian '53
Louis L. McIlhenny '36
R. C. Norberg '37
Samuel Robinson '36
Clarence R. Shook '48
Anna Schlorer Smith '44
Leon C. Sunstein '36
Harry C. Vitti '54

THERMOELECTRIC REFRIGERATION APPLICATION

BY
C. W. HARGENS[1]

An item that has excited considerable interest among visitors to the Laboratories recently is the use of thermoelectric refrigeration in a developmental instrument. The instrument, described in an earlier publication,[2] is for accurately determining the density of liquids.

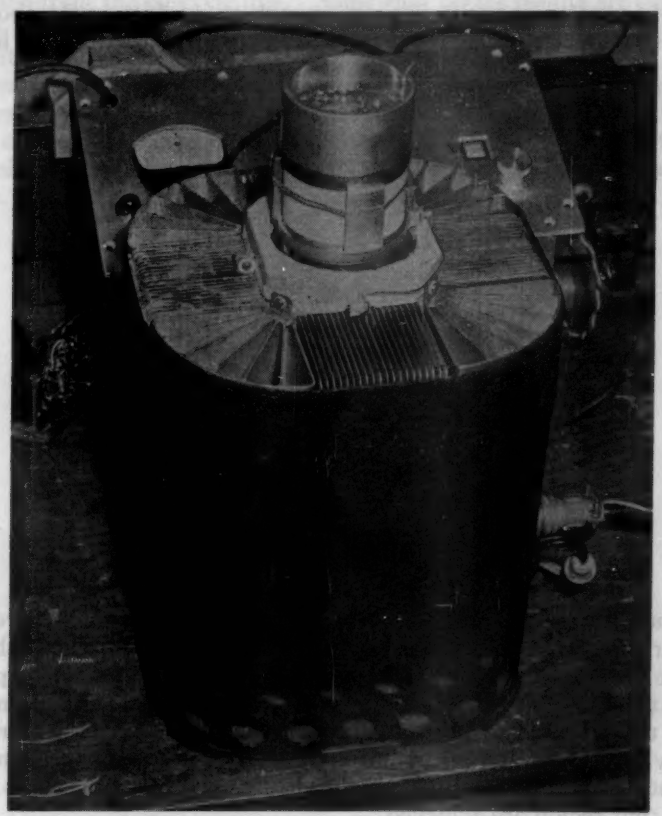

FIG. 1. Thermoelectric refrigerator for liquid density instrument.

[1] Senior Staff Engineer.

[2] C. W. Hargens, "Liquid Density Meter Development," JOUR. FRANKLIN INST., Vol. 262, p. 311 (1956).

EDITOR'S NOTE: A more detailed paper by Mr. Hargens on this subject appears in *Electronics*, Vol. 31, No. 49, Dec. 5, 1958.

Proofing whiskey is one specific application. The thermoelectric components in this case cool the liquid to a standard 60° F. and control to ±0.1° F.

Figure 1 shows the first experimental model of the cooling device designed and built by the Whirlpool Corporation's Research Laboratories at St. Joseph, Michigan. The density measuring head is shown in place on top with its float (not visible) extending downward into the cooler's cup.

The cup contains about 150 cc. of liquid. This can be cooled from a room temperature of 80° F. to the control point in several minutes. The refrigeration capacity is great enough to produce ice in about 20 minutes under average room conditions, if the thermostat is disconnected.

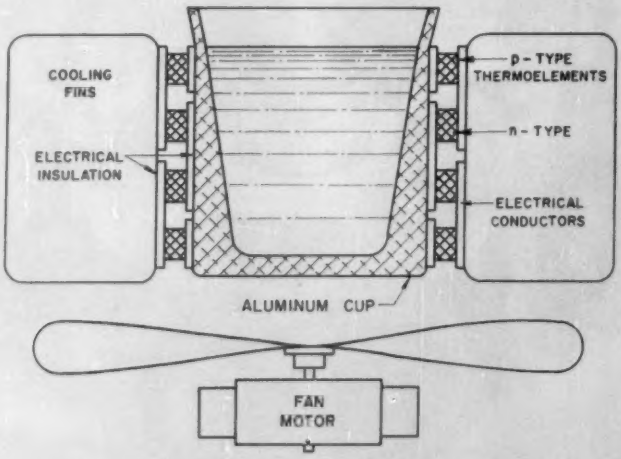

FIG. 2. Schematic diagram of thermoelectric refrigerator.

The principle of the thermoelectric refrigerator is the 100-year-old Peltier effect. Stated crudely, forcing direct current through a thermocouple can produce the absorption of heat at one junction and expulsion of heat at the other. The temperature of the absorption junction drops while the other rises. The refrigeration effect thus produced remained purely a laboratory novelty until recent solid state advances in semiconductors yielded practical thermoelements. Now efficiencies in the 5 per cent region are achieved.

Although this low efficiency sounds like a poor competitor for mechanical refrigeration, there are applications, such as the one described here, where convenience outweighs this disadvantage. The total power input to the present cooler is approximately 40 watts, which is of little consequence compared to the service rendered. A much more com-

pact unit is to be constructed, and its size together with simplicity will constitute a design which could not otherwise be achieved.

Figure 2 indicates schematically the arrangement of the cup, thermoelements, and cooling fins. A small fan is included to urge air through the system. Each of the four sides of the central cooling chamber carries ten thermoelements. These are p- and n-type bismuth telluride junctions. All forty are connected in series and may carry approximately 20 amperes when the high cooling rate is called for. This current can be derived from a very simple power supply consisting of a bridge of silicon rectifiers and a small, high-current transformer operated from the 60-cycle line. Somewhat better results are obtained with filtered direct current, but the difficulty of smoothing the high current is not worth the added equipment complexity.

One may conclude that here is a very compact little refrigerator with essentially no moving parts except for a small fan. The absence of the conventional gas system at high-pressure, sealed components, and expensive parts makes many of us believe this principle has an important future.

BOOK REVIEWS

LOGIC MACHINES AND DIAGRAMS, by Martin Gardner. 155 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1958. Price, $5.00.

Mr. Gardner has provided in this short book a compact survey of mechanical devices that have been constructed to solve elementary problems in formal logic together with a description of certain diagrammatic methods of achieving the same end. His role is that of historian rather than that of critic, and his treatment is non-technical. In this respect his book is a pleasantly written and readable piece of work.

Mr. Gardner has done well to begin with an account of the *Ars Magna* of Ramon Lull (1232/1235?–1315). This energetic practitioner of the Bonaventurian doctrine that finite things are copies of originals existing in the divine mind went so far as to demonstrate his beliefs by constructing systems of superimposed circles carrying the inscriptions of certain fundamental concepts, so that when the circles are rotated with respect to each other all the relations of the inscribed concepts corresponding to the truths of religion might be obtained. Lull's mystical art amounts, of course, to little more than a set of *petitiones principii*, involving the assumption of the religious truths that he takes his circles to establish, yet it must not be dismissed summarily, if only for the influence it exerted upon Leibniz in his *Ars Combinatoria*.

The book continues with an exposition of logic diagrams in general, with particular reference to the methods of Hamilton, Peirce, Carroll, Macfarlane and Venn. These methods are designed to solve problems in Aristotelian syllogistic inference. The remainder of the book contains a discussion of an original network diagram for the propositional calculus, and three chapters devoted to the description of the Stanhope Demonstrator, Jevons's Logic Machine and Marquand's Machine amongst others. Chapter 7 contains a brief account of window card devices for exhibiting the conclusions of different syllogisms, and Chapter 8 aligns us with more recent developments in logic machines, namely, electrically operated devices. A final chapter concerns the future of logic machines.

Notwithstanding the modest claims made by the author in connection with his network diagram for the propositional calculus, the reviewer feels that little or no improvement can be effected in present methods of truth-value analysis. Nor does an improvement seem necessary. From his experience of teaching logic, the reviewer has found that students have no difficulty in manipulating truth tables and, as a consequence, encounter no difficulty in visualizing the formal structure of truth-value relations.

The more sophisticated reader of Mr. Gardner's book might be disappointed to find no discussion of Turing Machines. It seems that the final chapter might well have contained a brief description of these hypothetical devices, which although not machines in Mr. Gardner's sense of the word, have nevertheless had an important theoretical part to play in recent logical developments concerning effective computability by a machine.

There are useful notes at the end of each chapter, and the book is pleasantly set out and illustrated.

DONALD J. HILLMAN
The University of Pennsylvania

BIOLOGICAL TREATMENT OF SEWAGE AND INDUSTRIAL WASTES, VOLUME 2, ANAEROBIC DIGESTION AND SOLIDS-LIQUID SEPARATION, edited by Brother Joseph McCabe and W. W. Eckenfelder, Jr. 330 pages, illustrations, 6 × 9 in. New York, Reinhold Publishing Corp., 1958. Price, $11.50.

This second volume precedes an earlier one covering the principles of aerobic oxidation and aeration in treatment of municipal and industrial wastes. The chapters were originally presented as conference papers and

consequently the continuity is not as smooth as if this were the work of a single author. However, it is excellently edited. The authors have been leaders in the "reduction of the degree of empiricism" in this field and have attracted outstanding contributors. The book will be indispensable to researchers, designers, and students of industrial waste treatment. The book consists of three parts.

Part 1. Anaerobic Treatment. A historic review precedes two fundamental discussions of the underlying biochemical principles. The organism-habitat, enzymatic, metabolic-pathway and other aspects of anaerobic oxidation are summarized. Theoretical considerations of the methane-carbon dioxide balance in sludge digestion are presented. Both discussions note the need for future study of the complex biological-chemical reactions and interactions. A short treatise with the catchy title "A Bacterium Looks at Anaerobic Digestion" suggest that future development might lead to high rate anaerobic processes competitive with present day aerobic systems used in commercial fermentation.

Recently-developed techniques for high-rate sludge digestion are presented with a prediction that the practise will grow considerably in the near future. Gas diffusion and design and operating considerations are presented in separate sections. There are experience-based suggestions included which should be important to designers and operators alike.

A discussion of gas chromatography, conductivity and ORP is followed by a report on the study of the influence of radioactive isotopes. In this reviewer's opinion, these are two of the best contributions. Each is based upon experimental findings and liberal use of graphical results facilitates comprehension. The easy-to-take, introductory discussion of the principles of operation will be appreciated by the inexperienced reader. The researcher will find these papers to be thought-provoking if he is not already using these techniques. Four papers discuss applications to pea blancher wastes, vegetable oil, detergent, packing house and pulp and paper wastes. The final paper deals with chemical and hydraulic characteristics of the anaerobic contact process.

Part 2. Sedimentation and Flotation. This part of the book concerns the use of conventional sedimentation and the newer flotation systems of separating solids from waste water.

Sedimentation is described by mode: overflow and free settling; zone settling and compression. The importance of flocculation rate detention and settling in basin design is emphasized. Laboratory procedures for accumulation of basic data are integrated with a discussion of extrapolation to plant design and performance. Hydraulic characteristics are treated in three chapters.

Flotation principles and applications with comparison to sedimentation are the basis of three chapters. One chapter treats the mathematical principles of flotation and is provided to serve as a basis for further research.

Part 3.—Vacuum Filtration and Sludge Conditioning. Sludge elutriation (solids washing) is discussed as a four-step-process which reduces coagulant demand by adjustment of alkalinity and freeing entrained gases. Elutriation methods are reviewed. Laboratory methods of determining sludge specific resistance and compressibility are given in one chapter which is supported by a mathematical treatment of filtration theory. This excellent discussion is adequately supported by tabular and graphical information. Two final chapters discuss practical applications of sludge dewatering.

ROBERT A. BAKER
The Franklin Institute Laboratories

CONTROL SYSTEM COMPONENTS, by John E. Gibson and Franz B. Tuteur. 493 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1958. Price, $12.00.

The servo engineer will find *Control System Components* a very welcome addition to his library. The book's major contribution is the derivation of the transfer functions of the most important building blocks in the control field. However, the authors have also chosen to discuss some of the most distressing problems of the design engineer. The subject matter is divided into thirteen self-contained chapters: seven deal with electrical components; two each are devoted to mechanical, hydraulic and pneumatic components.

The first chapter introduces the basic frequency response method of analysis used throughout the text, sandwiched between a discussion of basic electrical circuit components, and techniques for synthesizing RC networks. The remainder of the book can be read in any order. The subject of d-c. amplifiers is given careful attention in Chapter 2. The transfer functions of various types of phase inverters, cathode followers, differential amplifiers and chopper-stabilized amplifiers are derived. The drift problem is thoroughly discussed. Power amplifiers using vacuum tubes and transistors is covered generally in Chapter Three, with a special detailed discussion of thyratron amplifiers. There is also a good basic treatment of magnetic amplifiers and a brief analysis of a relay-type amplifier. Rotating machinery is presented in two chapters. Chapter 4 deals with d-c motors and generators, and Chapter 7 covers a-c. servomotors and induction motors. Very useful transfer functions are derived for many of these machines including the Amplidyne, the Rototrol and the Regulex. Although similar derivations are found in other texts, they are presented here in their most useful form to the servo engineer. The characteristics of various input-output devices including synchros and variable reluctance tranducers are compared in Chapter 5. Sources of synchro errors are thoroughly discussed. Chapter 6 describes the most frequently used phase-sensitive demodulator circuits, and compares their relative merits.

Recognizing that an increasing amount of servo work is being done in the non-electrical fields, the authors have devoted the remaining six chapters to these facets of the servo art. Mechanical components such as springs, dashpots and inertias are compared to their electrical analogues in Chapter 8. A very useful table of mechanical equalizer networks is also included in this chapter. Chapter 9 describes a range of specific mechanical components. Considerable emphasis is placed on the Gyroscope, but careful attention is given to strain gauges, the flyweight tachometer, the Universal joint and the differential for algebraic summation of shaft positions. The next two chapters are concerned with hydraulic systems. Chapter 10 deals with the pump-controlled system and Chapter 11 covers the valve-controlled system. Various types of pumps and servo valves are described and compared. The transfer functions of the two basic systems are derived. This treatment of the subject is not generally found in textbook form, so it should prove very useful. The final two chapters deal with pneumatic systems. The subject is introduced in Chapter 12 with a discussion of pneumatic flow and compressibility. Pneumatic-electric analogues are presented, and several control functions are tabulated. In the last chapter, some pneumatic components are described. Among these are pneumatic power supplies, control valves, motors and actuators.

Generally, the entire book is clearly written, and with the possible exception of Chapter 1, is concisely arranged. It should be a handy reference source for servo engineers.

MORTON SCHWARTZ
The Franklin Institute Laboratories

THE NEUTRINO, by James S. Allen. 168 pages, diagrams, $6 \times 9\frac{1}{4}$ in. Princeton, Princeton University Press, 1958. Price, $4.50.

"They seek him here, they seek him there. . . ." Since its existence was postulated by Pauli in 1933, the neutrino has given physicists little cause to doubt its reality, although by its very nature it defied direct detection until very recently, and its properties had to be deduced from indirect experimental evidence. Actually, this generally-accepted member of the family of fundamental particles of modern physics served essentially as a respectable "bugger-factor" to which was assigned the role of carrying away missing energy, momentum, and spin in the β-decay process, in accordance with the inviolate conservation laws. This mission could be accomplished by the emission of a neutral particle with essentially zero mass, which was virtually undetectable with the techniques available until very recently. The search for a direct interaction of the neutrino extended over a period of 29 years, culminating in the successful observation by Cowan *et al* of the inverse β-decay process, the only action of free neutrinos which can be predicted with certainty.

The first successful experiment to yield indirect evidence for the existence of the neutrino, (based upon the orbital electron capture process), was performed by the author of this excellent short treatise. Professor Allen has remained an active contributor in this field, and his unquestionable competence to discuss the subject is evident throughout the text.

In the present era when it is virtually impossible for a physicist to keep abreast of the literature even in his own particular area of research, it is indeed refreshing to find in a single book complete coverage of its field. There is a comprehensive, critical description of practically all of the experiments which have been performed, and an adequate theoretical background to permit an understanding of the physical principles involved.

Although the general subject had appeared to be closed in 1955, the Nobel Prize winning contribution of Lee and Yang in 1956 relating to the non-conservation of parity in weak interactions stimulated a large number of theoretical investigations and additional experimental studies of processes involving the neutrino and the anti-neutrino. Through the medium of notes added in proof, developments through March 1958 are included, although the original manuscript was completed in May, 1957.

Chapter I is a very brief introduction to the properties of neutrinos and the principles underlying the methods of detection. The rest mass of the neutrino, as determined by β-ray experiments, is discussed in detail in Chapter II, whereas Chapter III describes orbital electron capture experiments. Chapters IV and V deal with electron-neutrino angular correlation in β-decay from the theoretical and experimental points of view, respectively. The phenomenon of double β-decay, which provides a means of investigating the nature of the neutrino not revealed by β-ray spectra studies, is discussed in Chapter VI. Attempts to detect the free neutrino are described in Chapter VII, and the final chapter is devoted to neutrinos emitted or absorbed in processes involving mesons.

The book is well-organized, amply and clearly illustrated, and includes a list of references at the conclusion of each chapter. For each type of experimental approach, tables summarizing the essential features of the cited individual references are presented.

In view of the new revolutionary developments in this field, it is likely that active research will be continued for some time, and, as the author states, this monograph should be considered as a progress report. It is regrettable that so few phases of modern physics seem to be amenable to similar treatment.

MARTIN A. POMERANTZ
Bartol Research Foundation

LANDING GEAR DESIGN, by H. G. Conway. 342 pages, diagrams, plates, $6 \times 9\frac{1}{4}$ in. New York, The Macmillan Co., 1958. Price, $12.00.

Aircraft designers will find in this book an excellent review of the state of the art landing gear design accumulated over a fifteen-year period. The third volume in a series produced by the Royal Aeronautical Society for student aeronautical engineers, *Landing Gear Design* is practical and thorough. Two types of chapters may be found: those dealing with the general form of a landing gear system, and those dealing with specific parts of the system, such as tires, brakes, wheels and shock absorbers. Each type of gear is considered with numerous examples of the basic type and its variations being given. In each case the advantages and disadvantages are discussed and consideration given the conditions under which a given gear type is indicated.

The book is profusely illustrated with photographs, line drawings and perspective drawings showing applications of various gear layouts and component assemblies. The diagrams are especially good in the chapters entitled "General Layout of the Landing Gear," "Shock Absorbers," and "Retraction." Design details are given to such fine points as the durometer hardness most usual in rubber seals and glands, and surface finishes required for moving parts.

The thoroughness of the book may be understood by the fact that illustrative examples have been drawn from the aircraft of Canada, England, France, the United States and Italy. Possible developments

* Including attempts to account for the continuous spectra of β-ray energies before the neutrino was christened as such.

and present experimental gear are discussed in a chapter entitled "Unorthodox Landing Gear."

Two points in the book could be expanded to good advantage. In the chapter covering wheel design, one factor to be considered is stated to be the designing to resist fatigue, but insufficient discussion is given this point. In the chapter on stressing of landgear, several examples of solutions of specific problems would be helpful.

Over-all, this book, with its wealth of background and detail, will be a valuable addition to the engineering reference library.

RICHARD H. HOLLINGER
The Franklin Institute Laboratories

SOLID STATE PHYSICS, ADVANCES IN RESEARCH AND APPLICATIONS, VOL. 6, edited by F. Seitz and D. Turnbull. 429 pages, diagrams, 6 × 9 in. New York, Academic Press, Inc., 1958. Price, $12.00.

The sixth volume of the Solid State Physics Series, contains a greater diversity of subjects than the preceding ones. For this reason we shall give a brief review of each of the seven topics included. We shall state, however, at the outset, that the general standards are kept on a high level of quality and and thoroughness.

1. *Compression of Solids by Strong Shockwaves*—M. H. Rice, R. A. McQueen, T. M. Walsh pp. 1–65—Deals with the effect on Solids of Shockwaves induced by extremely high pressure detonation waves. The basic relevant concepts of fluid dynamics are outlined and followed by a review of experimental procedures. An extremely useful summary of presently published data is given. One would have liked a greater stress on the basic aspects of this specialized topic, in particular the connection with Bridgman's work, and a tentative atomic interpretation.

2. *Changes of States of Simple Solid and Liquid Metals.*—A. Borelius pp. 65–94—The fundamental question of the equation of state for condensed phases is examined by the most outstanding investigator in this field. Among the major aspects considered let us mention the relationship between structural entropy and structural energy, melting as an order disorder transformation, compressibility of cubic metals and the activation energy for self diffusion.

3. *Electroluminescence*—W. W. Piper and F. E. Williams, pp. 96–173—This important subject is methodically introduced, and the definitions clearly stated at the outset. After a general background on semiconductors and dielectric breakdown, the basic mechanisms of electroluminescence are presented and discussed, namely: Excitation process, energy transfer processes and emission processes. Considerable attention is then given to Zinc Sulfide. Other electroluminescence materials are briefly examined. The authors are to be commended for a clear, logical and thorough exposition.

4. *Macroscopic Symmetry and Properties of Crystals*—Charles S. Smith pp. 175–249—This is a systematic treatment of the symmetry properties of crystals. After an introduction to the relations between the symmetry of crystals and their properties, tensors of only one, two, three and four are examined. For each of these tensors corresponding outstanding properties of crystal are reviewed. A large area of the Physics of Crystals is thus covered with a stress on the tensor character of the properties. The content of this article combines naturally with Koster's article on space groups, in Vol. V of SSP.

5. *Secondary Electron Emission*—A. T. Dekker pp. 251–312—This article deals with electron emission induced by electron bombardment, a very important and very complex problem. First given is the experimental data on the yield of various solids: metals, semiconductors and insulators. Backscattering, reflection and characteristic energy losses of primaries are then discussed. A clear exposé is given of the various theories proposed to elucidate this phenomenon. It appears that much remains to be done before a satisfactory understanding of secondary emission may be reached.

6. *Optical Properties of Metals*—M. Parker Givens, 313–352—This classical subject is given an up-to-date and competent review. After a few pages devoted to the electromagnetic background, the author describes the various classical theories (Hagens-Rubens, Drude, Zener). He then discusses the internal photo-electric effect and the anomalous skin effect. The second half of the article is devoted to experimental methods and results.

7. Theory of the Optica lProperties of Imperfections in Nonmetals—D. L. Dexter, pp. 355–411—This is a theoretical account of the current state of understanding of the optical properties, with the presentation of the important equations and formalisms utilized in this field. In a sense it completes and brings up to date the 1954 review by Seitz (R.M.P. **26**, 7, 1954).

HENRI AMAR
The Franklin Institute Laboratories

THE SOLID STATE FOR ENGINEERS, by Maurice J. Sinnott, 522 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1958. Price, $12.50.

The variety of materials and processes with which today's engineers work requires such a broad knowledge of the physical sciences that a grasp of the basic principles of the factors controlling the structure and properties of materials is a prime necessity. This necessity has been observed for some time by Professor Sinnott and prompted this publication. As the author stated, he is attempting to bridge the gap between a textbook on engineering materials and one on solid state physics. He has presented the subject matter in 21 chapters based upon generally accepted principles of structure, properties, and energy relationships.

Each chapter is supplemented with a bibliography directed to the more detailed and basic considerations of the subject. Illustrative problems in each section are very helpful in learning the principles involved.

Some explanations of broad theories as presented by the author are subject to different interpretations and thus controversial, however, the intent was to give the student a working theory. An example of this is the comment that "the purer the metal, the lower its strength."

In attempting to cover such a broad field as the solid state in one volume, the author necessarily had to be brief and arbitrary. The last section on miscellaneous surface phenomena is much too brief. Possibly at this stage the author was exhausted. However, the importance of this extensive subject is becoming so great that it is now the basis for an International Congress of Catalysis and an International Conference on Surface Activity.

In general the topics are presented in a logical development and in a style that is easy to read. The book certainly fulfills the author's intent to give the student engineer an introduction to the many facets of the solid state. The practicing engineer who wishes to develop a more fundamental understanding of factors affecting the properties of materials can benefit from its study and will also find the volume a useful addition to his library.

R. S. DALTER
The Franklin Institute Laboratories

STRUCTURE AND EVOLUTION OF THE STARS, by Martin Schwarzschild. 296 pages, diagrams, 6 × 9 in. Princeton, Princeton University Press, 1958. Price, $6.00.

This book fills a critical need in modern astronomical literature. Summarized in it are many of the stimulating developments which have so vastly changed the study of stellar interiors. These developments originated with the introduction of nuclear physics into astronomy and much of the work has been practicable only because of the existence of the large electronic computers.

The book is divided into eight chapters. The first describes the observational foundation on which the theoretical work is based and which furnishes the tests for it. Most of this work is the product of the last decade and was made possible chiefly by the development of the multiplier photocell which has made possible precise measurements of faint stars.

The second chapter is concerned with the basic physical laws applied in the study of the state of stellar interiors. Topics considered are hydrostatic and thermal equilibrium, radiative and convective energy transport, the equation of state, opacity, the appropriate nuclear reactions, and conditions of the surface layers. The third chapter describes the mathematical techniques needed for the construction of stellar models concluding with the description of numerical integration.

The following four chapters describe in considerable detail the methods of constructing various stellar models. Chapter four deals with the stars in their initial state

after thermonuclear reactions have become the source of energy generation. For this purpose, the stars are divided into "upper main sequence," "lower main sequence," and subdwarfs. The first division is made because of the difference in the energy liberating process in the two cases and also because of the necessity of dealing with deep convection zones in the latter case. The apsidal motion test is developed in some detail and the chapter concludes with a description of the pre-main sequence contraction.

The fifth and sixth chapters deal with the models of stars in the early and advanced evolutionary phases respectively. Chapter seven is concerned with the final state when the stars presumably have become white dwarfs. The last chapter summarizes the conclusions that can be drawn from the model sequences and describes the story of stellar evolution as seen at present.

This book is not for the amateur, and there are portions which even the professional not working in the field of stellar interiors may not wish to read in full detail. However, the construction is such that the first and last chapters can be read with profit without the necessity of following all the intervening discussion. Certain topics—for example, the evolution of close binary systems—are deliberately omitted, either because they have been adequately covered elsewhere or because the current state of our knowledge is in too great state of change to make their inclusion suitable.

This work performs an outstanding service in gathering in one volume an authoratative summary of work scattered widely throughout the literature. The treatment is rigorous but the author never loses sight of the basic physical principles so that, even in the details of model construction, the reader can keep in view the basic considerations governing the computations. The only criticism that can be made is that the style is so clear and the descriptions so lucid that the casual reader, despite explicit warnings, may well take away the impression that in all phases of stellar evolution we are on firmer ground than in reality we are.

FRANK BRADSHAW WOOD
The University of Pennsylvania

CALCULUS OF VARIATIONS AND ITS APPLICATION, Proceedings of the Eighth Symposium in Applied Mathematics of the American Mathematical Society, edited by Lawrence M. Graves. 153 pages, graphs, $6\frac{1}{4} \times 10$ in. New York, McGraw-Hill Book Co., Inc., 1958. Price, $7.50.

This volume, the eighth in a series of proceedings of symposia in applied mathematics, continues in the tradition established by its seven predecessors. It contains the papers presented at the Eighth Symposium in Applied Mathematics, sponsored jointly by the American Mathematical Society and the Office of Ordnance Research on April 12–13, 1956, at the University of Chicago. Topics discussed by some of today's outstanding mathematicians include Elasticity, Plasticity, Dynamic Programing, Hydrodynamics, Diffraction, Conformal Mapping, Functional Analysis and Bounds for Eigenvalues.

Eric Reissner, in his paper, "On Variational Principles in Elasticity," demonstrates that Green's minimum principle for displacement and Castigliano's maximum principle for stresses are both direct consequences of a more general principle for stresses and displacements.

An interesting paper by D. C. Drucker, "Variational Principles in the Mathematical Theory of Plasticity," considers Perfect-plasticity theory and both the incrementally linear and the incrementally non-linear theories for work-hardening materials. Counterparts of the minimum-potential-energy and minimum-complementary-energy theorems are derived for stress strain relations of great generality. Absolute-minimum principles rather than relative ones are established. Commenting on Professor Drucker's paper, P. G. Hodge, Jr., in a special note, discusses the difference between two viewpoints, *viz.*, minimum principles and variational principles with respect to the physical sciences. In a similar note, H. F. Weinberger comments on the paper "Stationary Principles for Forced Vibrations in Elasticity and Electromagnetism" by J. L. Synge.

M. M. Schiffer discusses the technique of variational methods in conformal mapping indicating its usefulness in existence problems and extremum problems. The prolific Rich-

ard Bellman considers several problems in Mathematical Economics, particularly the application of Dynamic Programming to variational problems in allocation and smoothing processes.

"Variational Methods in Hydrodynamics" is a brief paper by S. Chandrasekhar in which he describes two examples of solving characteristic-value problems of high order by use of variational methods. J. B. Diaz presents an account of the underlying ideas of two fundamental methods for eigenvalue approximation due to Rayleigh, Ritz & Poincaré (upper bounds) and A. Weinstein (lower bounds). Diaz has attempted to fashion this account in a manner as self-contained as possible. Finally, E. H. Rothe relates some applications of Functional Analysis to the Calculus of Variations and Joseph B. Keller, in a lengthy paper, presents an extension of the laws of geometrical optics so as to include the effects of diffraction.

Most of the papers presented contain an extensive bibliography of related works. It must be emphasized that this volume is not for the beginner, either in the Calculus of Variations or the various fields to which its principles have been applied. The book is a compilation of the work of experts, most of whom have made full use of referencing the bibliography at frequent intervals in the text. This is great for "covering ground" but it doesn't do much in the way of "instructing the ignorant." The book, however, is highly recommended as an authoritative account of recent work by men renowned in their field.

LAURENCE MCGINN
The Franklin Institute Laboratories

AIRCRAFT AND MISSILE PROPULSION, VOLUME 1, by M. J. Zucrow. 538 pages, illustrations, 6 × 9 in. New York, John Wiley & Sons, Inc., 1958. Price, $11.50.

The first of a three-volume series to serve as an engineering text for the field of jet propulsion quite naturally deals with the thermodynamics of fluid flow. The presentation is made in five chapters beginning with a review of fundamental principles and moving through characteristics of propulsion systems, thermodynamics of compressible fluid flow, flow through nozzles, and flow through diffusers. Each chapter begins with a listing of symbols used throughout the chapter; a point which helps orient the reader for the rain of equations necessary in any discussion of thermodynamics. Unlike many texts, the equations developed to describe the various points are illuminated by numerous example problems. A separate section of problems and answers for each chapter allows the reader to keep a running check on his grasp of the contents.

While this series is drawn up for those who are not propulsion specialists, it would be well for any reader to spend a judicious amount of time in the first chapter of review. This would be especially true for those trained in scientific fields which utilize the metric system almost exclusively. It is also desirable for such readers to select several of the references listed after each chapter for some introductory reading.

Though unnecessary, it would be interesting to see photographs in addition to drawings in the chapters dealing with nozzles and diffusers. This would serve to illustrate actual applications of the various types.

In view of the announced shortage of engineers trained in the missile and high speed aircraft field, the introduction of this series of texts is a step in the right direction. The writer is well qualified to present this series both from the point of experience and as a teacher. The reader will find that this is not a book to be read through but rather to be worked through. The results will be gratifying.

RICHARD H. HOLLINGER
The Franklin Institute Laboratories

SPACE FLIGHT, by Carsbie C. Adams. 373 pages, illustrations, 5¼ × 8 in. New York, McGraw-Hill Book Co., Inc., 1958. Price, $6.50.

The sub-title of the book is *Satellites, Space Ships, Space Stations,* and *Space Travel Explained.* The sub-title is an accurate description of this fascinating new book on space travel.

This book by Dr. Carsbie C. Adams and his collaborators in the National Research and Development Corporation is a major improvement over the current run of books appearing on this subject. Unlike the others, this is destined to become a classic in the field. While this reviewer could indicate

precisely why, the reader is referred to the foreword written by Dr. Wernher von Braun who eloquently establishes this point.

There are logic and order in the format of the book. The first three chapters fashion a backdrop for what follows. The historical settings, the rocket and the theory of the rocket are covered here. The next three chapters deal with satellites. After presenting the satellite theory the current satellites from the Sputniks to Vanguard and Explorer are described. To this reviewer this is one of the highlights of the book, for this material has been presented fully and understandably, with authority and balance.

One chapter is devoted to the problem of escape using the basic equations necessary for full comprehension. The next chapter, on the space ship, constitutes a brilliant analysis of the problem of putting a vehicle into space. The various types of space ships are outlined and the effect of a human payload on the design of the space ship are investigated. There is a chapter on "The Architecture of the Universe" which is really a description of the solar system. The justification for the word universe is found in the last three pages.

Chapter 10 defines the role of the human being in space flight. The hazards of space and the equipment necessary are delineated. As in the chapter on the space ship this is brilliantly written demonstrating the complete competence of the author. The most important facet of communications is the next chapter. Incorporated in this is a section on transponders. Here is a new concept which will become more prominent as time goes on. The author paints a picture of global communications as a result of this development.

With the moon rocket destined to play a major role in the next few years there is altogether too little said about this feature. Perhaps it was intentional on the part of the author to underplay this subject because it may have been impossible to do it justice in the limited space available.

Navigation and interstellar flight are the topics of the next two chapters. Again these represent masterful presentations. The final chapter is a philosophical one and contains a magnificent quotation from Ray Bradbury's book "The Strawberry Window." If anyone asks why go into space, here is the answer.

This reviewer views this book with some regret. Here is one of the best written and authoritative books on the subject to appear in print, yet the chances of its becoming a best seller or even going into several editions is small. For some reason it is not the content which is the measure of the book's success. The success is measured by the prominence of the author and the promotion which goes into it. Thus far this reviewer has seen little promotion and the name of Adams is relatively unknown in space travel circles. The combination of the two will possibly doom the book as far as the general public is concerned. There should be some way in which books of this caliber could be brought to the attention of the widest possible audience.

I. M. LEVITT
Fels Planetarium of The Franklin Institute

STOICHIOMETRY FOR CHEMICAL ENGINEERS, by Edwin T. Williams and R. Curtis Johnson. 350 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1958. Price, $8.00.

This volume is intended for beginning students in chemical engineering. The authors state that theirs was a threefold purpose: (1) stress fundamentals, (2) inculate neat and logical habits of problem solution, and (3) preview unit operations. They have effectively accomplished their purpose.

The subject matter is logically developed in clear, easy-to-understand style. Liberal use of examples facilitates comprehension. In some cases several alternate solutions are presented with a discussion of the reasoning used in selecting a basis and system of solution. Problems follow each chapter with answers in a separate appendix. A unique feature is that some of the answers include suggested basis of solution and key steps. The student or the practising engineer wishing review of the subject matter will find this helpful. The appendices are especially thorough and consist of tabular and graphical presentation of conversion factors and basic chemical and thermodynamic data. This section alone will provide a useful reference.

The subject matter is introduced by chapters on mathematical and chemical fundamentals. These lead to separate treatments of material and energy balances. The sections on heat capacities and thermochemistry are especially well elaborated. The concept of unit operations is introduced with discussions of evaporation, humidification and distillation. Unit processes are characterized by studies of combustion and sulfur oxidation. Finally a realistic detailed description of the development of a vinyl acetate process is given wherein the role of stoichiometric principles is demonstrated.

This text is excellent. If it has a fault it is one of omission and brevity. Certain topics that are important in future chemical engineering training that get little or no treatment are crystallization, adsorption and the use of ternary equilibrium data, critical data and reduced states. Subsequent editions would be improved by such expansion.

ROBERT A. BAKER
The Franklin Institute Laboratories

PUBLICATIONS RECEIVED

ELECTRONIC ENGINEER'S REFERENCE BOOK, edited by L. E. C. Hughes. 1311 pages, diagrams, 4¾ × 7¼ in. New York, The Macmillan Co., 1958. Price, $18.00.

METALLURGICAL THERMOCHEMISTRY, by O. Kubaschewski and E. Ll. Evans. Third edition (revised), 426 pages, diagrams, 5½ × 8½ in. New York and London, Pergamon Press Inc., 1958. Price, $10.00.

FUNDAMENTALS OF RADIO AND ELECTRONICS, edited by William L. Everitt. Second edition (completely rewritten), 805 pages, 6 × 9 in. Englewood Cliffs (N. J.), Prentice-Hall, Inc., 1958. Price, $11.00.

CHAMBERS'S TECHNICAL DICTIONARY, edited by C. F. Tweney and L. E. C. Hughes. Third edition (revised), 1028 pages, 5½ × 8¼ in. New York, The Macmillan Co., 1958. Price, $7.50.

MECHANICAL VIBRATION, by G. W. Van Santen. 310 pages, diagrams, 6 × 9 in. New York, The Macmillan Co., 1958. Price, $8.50.

REVIEW OF COLLEGE PHYSICS. VOLUME II. HEAT, THERMODYNAMICS, KINETIC THEORY, ELECTRICITY AND MAGNETISM, by Robert L. Weber. 158 pages, diagrams, 6 × 9 in. Englewood Cliffs (N. J.), Prentice-Hall, Inc., 1958. Price, $2.25 (paper).

REVIEW OF COLLEGE PHYSICS. VOLUME III. LIGHT, ATOMIC AND NUCLEAR PHYSICS, by Robert L. Weber. 150 pages, diagrams, 6 × 9 in. Englewood Cliffs (N. J.), Prentice-Hall, Inc., 1958. Price, $2.25 (paper).

INTRODUCTION TO MODERN PHYSICS, by C. H. Blanchard, C. R. Burnett, R. G. Stoner and R. L. Weber. 414 pages, diagrams, 6 × 9 in. Englewood Cliffs (N. J.), Prentice-Hall, Inc. 1958. Price, $10.00.

INFLUENCE OF HOT-WORKING CONDITIONS ON HIGH-TEMPERATURE PROPERTIES OF A HEAT-RESISTANT ALLOY, by John F. Ewing and J. W. Freeman. NACA Report 1341, 52 pages, illustrations, 9¼ × 11¾ in. Washington, U. S. Government Printing Office, 1958. Copies obtainable from NACA, Washington.

PRACTICE IN THINKING. A LABORATORY COURSE IN INTRODUCTORY CHEMISTRY, by Jay A. Young. 92 pages, diagrams, 8½ × 11 in. Englewood Cliffs (N. J.), Prentice-Hall, Inc., 1958. Price, $2.95 (paper).

ADVANCES IN STEEL TECHNOLOGY IN 1956. United Nations Publication. 107 pages, illustrations, 8½ × 11 in. New York, Columbia University Press (distribution agent in U. S.), 1958. Price, $1.00 (paper).

INTRODUCTION TO HISTORICAL GEOLOGY, by Raymond C. Moore. Second edition, completely revised, 656 pages, illustrations, 7½ × 10 in. New York, McGraw-Hill Book Co., Inc., 1958. Price, $7.95.

BOOK NOTES

TECHNOLOGY OF COLUMBIUM (NIOBIUM), edited by B. W. Gonser and E. M. Sherwood. 120 pages, illustrations, 8½ × 11 in. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Limited; 1958. Price, $7.00.

A monograph of seventeen papers presented at a symposium on columbium (niobium) by the Electrochemical Society, held in Washington, D. C., May 12–16, 1957. The sources, economic aspects, and physical properties of columbium are covered in the first three papers. Three papers deal with the extraction and preparation of the pure metal and two with analytic techniques. Oxidation and mechanical properties are the subjects of three papers each. The remaining topics covered are the use of columbium in reactors, electroplated metals on columbium, and vacuum reactions during sintering. The choice of material is good, and the subject is timely. The text is well illustrated and referenced.

HANDBOOK OF AUTOMATION, COMPUTATION AND CONTROL. VOLUME 1, CONTROL FUNDAMENTALS, edited by Eugene M. Grabbe, Simon Ramo and Dean E. Wooldridge, v.p., diagrams, 6 × 9 in. New York. John Wiley & Sons, Inc., 1958. Price, $17.00.

This three-volume handbook has been prepared to bring order out of the chaos of existing literature on the general subject of automation and control. It is meant to give technologists a unified work upon which they may depend for many years. If the succeeding volumes meet the standard set by this first one, the aim will be fulfilled. The editors, recognized experts themselves, have solicited the help of 104 specialists in preparing the handbook. The result is a thoroughly competent and useful work.

Volume 1, on Control Fundamentals, covers five main topics: General Mathematics (including conformal mapping, probability, statistics, etc.); Numerical Analysis; Operations Research; Information Theory and Transmission; and Feedback Control (including sampled-data systems, nonlinear systems, etc.). The five main sections are further subdivided into twenty-six chapters (paged separately), each covering one phase of the subject and each having a list of references. A detailed table of contents makes it easy to locate information on the 177 sections comprising the subdivisions of the chapters.

In contrast to most handbooks, this one is printed on good paper, in type large enough to be read with ease.

MODERN CHEMICAL PROCESSES, VOLUME 5. 154 pages, illustrations, 8 × 11½ in. New York, Reinhold Publishing Corp., 1958. Price, $5.00.

This is the fifth volume in a series of reprinted articles which contains fourteen recently developed chemical processes, all now in operation.

INTRODUCTION TO THE DESIGN OF SERVOMECHANISMS, by John L. Bower and Peter M. Schultheiss. 510 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1958. Price, $13.00.

This book deals with topics covered in the servomechanisms program at Yale University during the past ten years. The mathematical level throughout is that of the undergraduate course. No previous background is assumed aside from a knowledge of linear circuit theory and some understanding of linear differential equations. The authors provide a systematic approach to design, dealing with the principal performance requirements, such as harmonic response, time response, error coefficients and noise response and giving attention to the common aspects of non-linear operation. A self contained volume including an appendix covering servomechanism components, enables the reader to follow examples used in the text and to work representative problems without resorting to outside references.

THEORY OF A. C. CIRCUITS, by Sylvan Fich and James L. Potter. 453 pages, diagrams, 6 × 9 in. Englewood Cliffs (N. J.), Prentice-Hall, Inc., 1958. Price, $11.35.

Through the author's detailed explanations, this text provides a comprehensive coverage of A. C. circuit theory and the basic concepts necessary for the study of modern network theory. Although the treatment is essentially mathematical, there are no prerequisites other than the conventional first courses in mathematics and physics. All other mathematical techniques, including the basic theory of determinants and matrices, are developed in the text. Also, many illustrative examples both numerical and symbolic are included.

PROBLEMS IN EUCLIDEAN SPACE. APPLICATION OF CONVEXITY, by H. G. Eggleston. 165 pages, diagrams, 5½ × 8 in. New York, London, Paris & Los Angeles, Pergamon Press, 1957. Price, $6.50.

This monograph, one of the International Series in Pure and Applied Mathematics, was the Adams prize essay of the University of Cambridge, 1955–1956. The text covers only those problems which concern convex sets in real Euclidean space of two or three dimensions. Ten problems are dealt with, divided into four chapters, in each of which convexity is progressively more dominating than in the previous chapter. The monograph is for advanced students and research personnel in pure mathematics, both of whom should find the material challenging.

THE THRESHOLD OF SPACE, edited by M. Zelikoff. 342 pages, illustrations, 7½ × 9½ in. New York and London, Pergamon Press Inc., 1957. Price, $15.00.

In 1956, the Air Force Cambridge Research Center sponsored a Conference on Chemical Aeronomy. Outstanding men in upper atmosphere research came from several countries to exchange ideas. This volume contains the 45 papers presented at that conference. Four main divisions cover the following fields: Atmospheric Photochemistry—Earth and Venus (9 papers); Spectroscopy and Photochemistry (19 papers); Rocket Probing of the Upper Atmosphere (6 papers); and Phenomena Produced by Hypersonic Flight (11 papers). Probably no one person can understand all the papers, but there is enough material on a given subject to warrant study by all workers involved in rockets, satellites, space travel and upper atmosphere research. The papers pose most of the unsolved problems (both theoretical and experimental) in space travel and also clarify the existing theories.

ECONOMIC OPERATION OF POWER SYSTEMS, by Leon K. Kirchmayer. 260 pages, illustrations, 6 × 9 in. New York, John Wiley & Sons, Inc., 1958. Price, $12.00.

"Electronic brain" methods utilized by electric utilities to promote economy in production, is the theme of this text. Application of these methods to power systems has effected notable monetary savings as experienced through accurate consideration of transmission losses. Emphasis is placed on theoretical developments and computer use in dealing with and solving system problems. How matrix methods are employed to derive transmission loss formulas, which in turn are the basis for computational procedures applied to computers is discussed. Proper use of analog and digital computers to obtain transmission loss formulas is also described. Additional features in this volume include mathematical models, analytical methods and computer applications to prediction and improvement of performance of systems.

A SCIENTIST OF TWO WORLDS: LOUIS AGASSIZ, by Catherine Owens Peare. 188 pages 5½ × 8¼ in. Philadelphia, J. B. Lippincott Co., 1958. Price, $3.00.

This absorbing biography of a great scientist, naturalist and teacher, Swiss born Louis Agassiz, presents a vivid picture of his dynamic personality and dedication to the study of natural history.

THE PHYSICAL FOUNDATION OF BIOLOGY, by Walter M. Elsasser. 219 pages, diagrams, 6 × 9¼ in. New York and London, Pergamon Press Inc., 1958. Price, $4.75.

The author, a theoretical physicist, lucidly discusses the question "What is the difference between an organism and an automaton?" Dr. Elsasser describes three recent develop-

ments in theoretical physics—theory of automata, information theory, and the theory of microscopic measurement in the atomic and molecular domain. The author believes that these developments justify a new approach to biological theory; he aims in this book to pose the problem in such a way that it will stimulate new work in the field. Physicists, chemists and biologists should find this a provocative invitation to re-examine old concepts.

ADVANCES IN APPLIED MECHANICS. VOLUME 5, edited by H. L. Dryden, Th. von Karman and G. Kuerti. 459 pages, diagrams, 6 × 9 in. New York, Academic Press Inc., 1958. Price, $12.00.

Volume 5 of this useful series contains reviews and surveys of the current state of research in seven subjects: supersonic air ejectors (35 pages); unsteady airfoil theory (55 pages); theory of distributions (20 pages); stress wave propagation in rods and beams (84 pages); problems in hydromagnetics (38 pages); mechanics of granular matter (74 pages); and condensation in supersonic and hypersonic wind tunnels (142 pages).

The excellence of the book is unquestionable—it provides authoritative "state of the art" reviews written by outstanding contributors on the most recent advances in the applied mechanics field. The complete bibliographies will be helpful to students, as well as to research men.

FREE RADICALS, by Francis Owen Rice. 278 pages, diagrams, 8½ × 11 in. Washington, The Catholic University of America Press, 1958. Price, $5.00 (cloth).

The author, who has contributed greatly to a better understanding of the chemistry of free radicals, has collected his contributions to the subject in this volume, as a convenience for himself and his students at The Catholic University of America. Beginning with his first paper, "Free Radical Decomposition of Hydrocarbons," published in 1931 in the *Jour. Am. Chem. Soc.*, the volume presents 60 papers in chronological order, ending with "Reaction of Methyl Radicals with Hydrogen Atoms," which appeared in the same journal in 1958.

The contents of the papers are well known to workers in the field, for many of the articles are now classics. The collection into a new volume of these contributions, many of which have been unobtainable, will be welcomed.

SOME PROBLEMS IN CHEMICAL KINETICS AND REACTIVITY. VOLUME 1, by N. N. Semenov. 239 pages, 6 × 9 in. Princeton, Princeton University Press, 1958. Price, $4.50 (paper).

This English translation (by Michel Boudart) of the second edition of a Russian work first published in 1954 presents a survey of Professor Semenov's fifteen years of intensive work in the field of chemical kinetics. This English version, made possible by the National Science Foundation, will be an invaluable aid to students entering the field. The subject matter is mainly devoted to radical and radical-chain reactions, for the author makes no attempt to cover the whole field of the reaction mechanism. The translation will be published in two volumes; the present Volume 1 contains the first two parts (one on "Radical Reactions" and one on "Chain Initiation and Termination") of the Russian work.

PROGRESS IN BIOPHYSICS AND BIOPHYSICAL CHEMISTRY. VOLUME 8, edited by J. A. V. Butler and B. Katz. 409 pages, diagrams, 6½ × 9¾ in. New York and London, Pergamon Press Inc., 1958. Price, $17.50.

The eighth volume in this excellent series of critical reviews in the field of biophysics contains nine papers, roughly divided into two main categories—four dealing with physiological problems and five dealing with physico-chemical processes. Three of the papers are by American authors; the others are by Englishmen.

The physiological papers cover hearing, human color vision, the axon surface and the ionic permeability of the red cell membrane. The physico-chemical articles deal with the electrochemistry of the bacterial surface, the effect of radiation on DNA synthesis in mammalian cells, the biosynthesis of some connective tissue components, the physical chemistry of deoxyribosenucleic acid, and the biosynthesis of protein. Each paper begins with a complete table of contents and ends with a comprehensive bibliography.

THE ENCYCLOPEDIA OF CHEMISTRY (SUPPLEMENT), edited by George L. Clark. 330 pages, 7 × 10 in. New York, Reinhold Publishing Corp., 1958. Price, $10.00.

Supplementing the original *Encyclopedia* (reviewed in the JOURNAL, Vol. 263, p. 475), this present volume adds more than 200 articles to those covered originally. It is the editors' intention to revise the original work, incorporating the supplement, as soon as it is economically feasible. Material in the supplement maintains the high standard of accuracy evident in the main work. The present volume emphasizes the area within which physics and chemistry can no longer be separated, namely, the chemistry of nuclear phenomena. Random samples are "Plasma," "Jet Propulsion," "Nuclear Magnetic Resonance," "Radiocarbon Determination," and "Radiation Biochemistry."

INTRODUCTION TO PHOTOGRAPHIC PRINCIPLES, by Lewis Larmore. 229 pages, illustrations, 6 × 9 in. Englewood Cliffs (N. J.), Prentice-Hall, Inc., 1958. Price, $8.00.

In this easily comprehensible volume which stresses the scientific concept of photography, the author shows the amateur photographer how to apply the basic principles of physics, optics and chemistry, to create photographs of excellent quality. By assuming no previous knowledge of the subject on the reader's part, the text begins with the various types of cameras, their capabilities and limitations. Succeeding chapters dwell upon optical problems including image intensity, film sensitivity, elements of photographic chemistry and printing. More advanced phases of optical theory and also the essentials of stereoscopic (3-D) photography, color photography and colorimetry are covered. An appendix presents fifteen laboratory experiments which can be conducted with modest facilities and equipment along with 100 illustrations. The really serious camera enthusiast will welcome this book for its lucid, easily understandable account of the scientific bases upon which photography rests.

FOURTH NATIONAL SYMPOSIUM ON VACUUM TECHNOLOGY TRANSACTIONS, edited by Wilfred G. Matheson. 176 pages, illustrations, 8¼ × 11 in. New York, London and Paris, Pergamon Press Inc., 1958. Price, $12.50.

Containing the papers presented at the 1957 symposium held in Boston by the American Vacuum Society, this fourth progress report on vacuum technology will be welcomed by workers in the field. Twenty-eight papers are given covering basically three fields of interest: scientific bases of vacuum techniques; methods for obtaining and measuring high vacuum and ultra high vacuum; and industrial applications. The importance of vacuum techniques in space research is evident in this latest volume.

CURRENT TOPICS

Master Flight Reference System.—Development of a master flight reference system which intergrates output for auto pilot, Doppler radar, low altitude bombing (LAB) maneuvers, dead-reckoning equipment and all-attitude indication has been announced by General Electric. The new aircraft all-attitude compass system—designated Type SR-1 by Instrument Department engineers—uses non-floated gyros with a drift rate of only one degree per hour. The gyros require no heaters, literally functioning on demand.

First application of the General Electric device is on the Navy's A3J Vigilante, a supersonic attack plane which rolled out at North American Aviation Inc., Columbus, Ohio, in mid-May.

A unique "Roll Yaw Computer" translates Earth's coordinates into aircraft coordinates, furnishing proper signals for weapons delivery and automatic flight control. The new computer also allows the use of late-type remote 3-axis all-attitude indicators.

Compensation for the Earth's rotation and east-west ground speed can be supplied to the system from Doppler radar or other computing devices. When such equipment is not available, corrections can be inserted manually on the Controller.

The east-west ground speed correction feature provides heading corrections for apparent drift caused by aircraft motion around the axis of the Earth.

Heart of the SR-1 is the 3-gyro, 4-gimbal reference platform. This component weighs only 24.5 lbs., a significant weight economy achieved with the use of non-floated gyros rather than heavier floated types.

Electronic deviation compensation is incorporated to provide SR-1-equipped aircraft with the best possible magnetic heading information.

The entire SR-1 system is transistorized and incorporates plug-in type modular construction for ease of maintenance. Only silicon transistors are used to permit operation over a wide temperature range.

Aircraft instrument design engineers responsible for the development of General Electric's SR-1 have contributed many well-known devices for critical aviation measurement. Prominent on the long list are the MA-1 gyro-compass system (used as the eyes of "Operation Deepfreeze" and the principal navigation device on the USMC Crusader's record-making "Project Bullet") and the mass fuel flowmeter used widely on military aircraft and commercial jetliners.

Airborne Television for Ice Reconnaissance.—Ice reconnaissance up to a radius of 35 miles of an icebreaker is now possible through the medium of airborne television, according to a joint U. S. Navy-Philco Corporation statement. On a recent trip of the icebreaker *U. S. S. Glacier* to Thule Air Force Base, Greenland, TV pictures of surrounding ice conditions were relayed instantly from a helicopter to the television monitors aboard the *Glacier*. Two TV monitors were used on the icebreaker—one on the bridge and the other in the combat information center.

Thus, the captain and his executive officer who are the only qualified ice observers aboard an icebreaker, received on-the-spot information of ice field conditions without leaving the ship.

Present ice reconnaissance is obtained by having a qualified ice observer scout the ice field from a helicopter and radio the information back to the icebreaker. Also, the reconnaissance area is limited to 10 miles due to the ever changing weather conditions in the Arctic.

Word descriptions of ice conditions and later briefings after the copter's return depends on the observer's ability to describe what he saw. This method is also time consuming. However, with a television monitor before him, the *Glacier's* observation officer was able to make a constant evaluation of ice conditions ahead as the ship continued on her course.

In the future, the Navy expects to install the airborne television equipment in a remote controlled helicopter thereby increasing the range of its ice reconnaissance and eliminating any possibility of loss of life due to the Arctic storms.

Equipment used in this Arctic experiment was developed by Philco Corporation's Government and Industrial Division and included a special airborne TV transmitter and Philco's high resolution industrial television camera.

According to Commander Huston, captain of the *Glacier*, the experiment was quite successful. He said the television pictures beamed from the helicopter were very sharp and clear.

Telltale signs in the ice, showing the path of least resistance through the heavily packed ice, were readily discernible on the TV monitors. Observers on the Glacier were able to distinguish between old and new ice, pack ice and pressure ridges in the ice field. Also, open patches of water and large moving ice-bergs were clearly seen on the monitors.

The Navy Department spokesman said the Philco UHF video transmitter, which was used in the Arctic test, operates in the 780–900 megacycle frequency range up to a distance of 35 miles. A wide angle lens was used on the Philco vidicon TV camera which is small enough to fit in a briefcase.

New Cameras for High Speed Laboratory Use.—Two laboratory standard instruments—rotating mirror and rotating drum cameras—to provide accurate and reliable photographic records of hypervelocity phenomena have been developed for sale by Avco Manufacturing Corporation's Research and Advanced Development Division, Wilmington, Massachusetts.

The devices are valuable in studies of radiation, explosive compound development, solid fuel ignition, propulsion, ballistics combustion, chemical recombination, plasma tunnels, magnetohydrodynamics, spark gap discharges, high speed electromechanical units such as relays, and other transient events which emit or reflect light.

Avco's rotating mirror and rotating drum cameras are being exhibited for the first time at the ISA instrument-automation conference in Philadelphia on Sept. 15–19. The units were developed and in use for two years in connection with company research in high temperature gas dynamics and atmospheric re-entry problems involved in the Air Force Titan intercontinental ballistic missile program.

The rotating mirror camera records position-versus-time relationships, sweeping the image from a hyper-

sonic object source onto fast emulsion film strips. Heart of the unit is a hexagonal, first surface mirror, rotating at 3000 r.p.s., which provides writing speed of 4 mm./microsecond for total writing time of 50 microseconds. Other components are a cast aluminum main housing, standard F 2.5 lens, two curved film holders and air turbine drive.

The rotating drum camera, in addition to making position-time film records, also may be used with a densitometer to produce accurate data of hypervelocity phenomena. In conjunction with a Schlieren system it will produce microsecond-order photographic frames for graphical analysis. A precision-manufactured, dynamically-balanced drum, rotating at 600 r.p.s. for writing speed of 0.19 mm./microsecond and total writing time of 1667 microseconds, distinguishes this second instrument. The fast emulsion film is fixed to the inner periphery of the drum.

Both cameras operate on compressed air, driven through shaped nozzles at turbines integral with either the rotating mirror or drum units. The advantages of only one moving part and one pre-operational adjustment—focus which is set and locked—make the Avco cameras inherently simple and reliable.

For data reduction, both cameras provide continuous optical records which are readily measurable. Constant focal distances and known rotation speeds of the mirror and drum eliminate the major variable factors for accurate calculation of velocities.

"Freon"-TF Solvent Used in Film-Drying.—Photographers who've been dipping their newly processed but still wet film in ethyl alcohol to speed drying for rush printing jobs can add safety to the process by using a blend of alcohol and "Freon"-TF solvent.

Advantage of the blend is that "Freon"-TF, in itself nonflammable, nontoxic, and nonexplosive, counteracts the flammability of the ethyl alcohol without significantly reducing the latter's ability to absorb moisture from the wet film. Moreover, du Pont chemists say, the lower concentration of ethyl alcohol in the blend reduces the possibility of film being adversely affected or attacked by the alcohol during the quick-dry treatment. Recommended blend is one made up of 60 per cent "Freon"-TF and 40 per cent anhydrous ethyl alcohol, by volume.

Ready mixed solutions of the blend of ethyl alcohol and du Pont "Freon"-TF solvent are available as "Freon" Printed Circuit Cleaner from John B. Moore Corp., Nutley, N. J. The du Pont Company manufactures only the "Freon"-TF solvent for bulk sales. Consumer-size packages of "Freon"-TF solvent alone are available from John B. Moore Corp., and from Wholesale Supply Company, Los Angeles, Calif.

High-Precision Gear Checker.—Precision checking the lead of gears has heretofore required great skill and laborious procedures with gage blocks, micrometers, verniers and gear trains. A new lead-measuring instrument, the 1218A Checker, developed by Michigan Tool Company, Detroit, does away with these complications and precision checks gear leads from zero to infinity So simple is the unit to set up that no special training is required of the operator. Calculations are virtually eliminated. The sine-bar controlled machine, although primarily designed for external and internal helical gears and spur

gears, also checks either right- or left-hand leads on herringbones or worms. The checking instrument has a capacity for gears up to 18 in. in diameter with shaft lengths to 24 in.

A built-in optical system is the basis of the new easy-setup method. A 56× macroscope with graduated eyepiece, providing an unreversed right-side up image, is built in over the sine-bar table. Exact-angle location is merely a matter of aligning cross-hair markings on the scope's eyepiece with the precision-graduated sine-bar plate. The scope and eyepiece divides a degree into 1000 parts. The sine-bar table is merely rotated manually from a handwheel and is worm gear set. The sine-bar plate is positively locked in position without the slightest deviation from the correct scope setting.

Combined with the optical system is an electronic gage for recording errors in tooth profile of external and internal spur and helical gears. This unit consists of a light-load gaging head and high-response amplifier connected to a calibrated meter to visibly indicate how far off lead the work may be in increments of 0.0001 in.

Also included as part of the 1218A Lead Checker is an integral recording unit to provide a visual record of all checking data. The recorder is mounted convenient to the operator for ready reference. It records, in ink, on Michigan-supplied paper printed in increments of 0.0002 in. which can be interpolated to 0.0001 in.

Michigan's lead measuring instrument can be operated from front or rear. The gear being checked is mounted between centers and the indicator is adjusted so that the contact point is normal to the face of the gear tooth. Turning the handwheel, glides the indicator contact along the face of the tooth (as the gear rotates in proper relationship due to the sine-bar control). Any deviation on the indicator shows change from specified lead.

The new checker is a horizontal machine built for maximum rigidity to obtain top precision. All operating mechanisms, including the sine bar, are completely enclosed against foreign matter and are tamper-proof. All adjustments are at convenient working height.

The machine has two tables—a sine-bar table and a table carrying the indicator head. Both tables move on ball races. The sine-bar mechanism synchronizes movement of the tables and work spindle in correct lead-checking relationship.

Compactness is also a feature of the machine—floor space requirements are less than 18 sq. ft. Machine height is only 4 ft.

Airborne Radar Used for Ground Mapping.—An airborne radar system that produces clear, detailed ground maps from behind a protective cover of darkness, clouds or fog has been developed by Philco Corporation. A company spokesman also said that Philco's Government and Industrial Division has received a contract to build eight of these new high resolution radar systems for the U. S. Army Signal Supply Agency. This all-weather radar equipment will be used in advanced drone reconnaissance aircraft.

Compact enough to be used in most operationsl aircraft, the lightweight radar system uses a new signal processing technique known as "REDAP."

Nuclear Emulsion Scanner.—University of Michigan physicists have combined a powerful microscope, a television camera, and a special elec-

tronic computer to form a machine that can recognize certain situations as well as humans can, and 15 times faster. Their device is known officially as "a nuclear emulsion scanner." But they call it "Terry," for the Greek "Thereteira," or "huntress." Terry's job is to detect and count the thousands of faint tracks left by atomic particles flying through thick sheets of special photographic film in U-M atom smasher experiments. It represents "the first real success among half a dozen attempts in the U. S. and Europe to solve this recognition problem instrumentally," says Terry's inventor, Prof. Paul V. C. Hough.

Terry holds great promise of relieving humans of the slow, tedious task of examining hundreds of strips of film with microscopes, Professor Hough says. The device can automatically scan as much film in one and one-half hours as a human can in four days. Furthermore, it does so with 100 per cent efficiency in all areas of the film, while a human observer's efficiency drops rapidly in areas of many tracks.

Terry's "eye" is the television camera. It looks through a 200-power microscope at a different tiny square of the film every 60th of a second. Her "brain" is a specialized electronic computer which occupies 50 square feet of space. It "remembers" dots within the squares that are dark enough and wide enough to be parts of a track; if it detects 15 consecutive dots within a square, it records them as one track. Terry's "hands" are an electronic printer which keeps a running count of the tracks, and a plotter which draws a graph showing the frequency of the tracks. Another feature of the machine is definitely inhuman, Professor Hough says: the results of Terry's "consideration" of a scene can be displayed on a television screen at any state of her analysis.

The film strips come from the University's cyclotron, where they are used to record the flight of atomic particles deflected by the nuclei of the atoms in small targets, and sorted by a magnetic field. The particles strike the film edge-on, leaving a faint line behind. By recording the number of particles flying through film placed at different angles from the target, the U-M physicists gather clues as to the structure of the nuclei of the target's atoms. Terry does this automatically.

As many as 30 of the film strips, representing more than six months of counting by a human, are used in just one experiment with the cyclotron, Professor Hough points out. The new scanner can process them in four to five days.

"The job of finding tracks with a microscope in a 12-inch strip of film is equivalent to locating a one-inch-long needle with a four-inch reading glass in a field 20 feet wide and 240 feet long," he notes.

Terry does her work by looking for a millionth of a second at each of 130 squares which extend across the width of the film. After one band of squares is examined, the carriage holding the film shifts to the next band, and the process is repeated.

Professor Hough reports that he and his associates are trying to adapt Terry to the "much more complicated" recognition problems in the study of high-energy physics' "strange particles" and of cosmic radiation from outer space.

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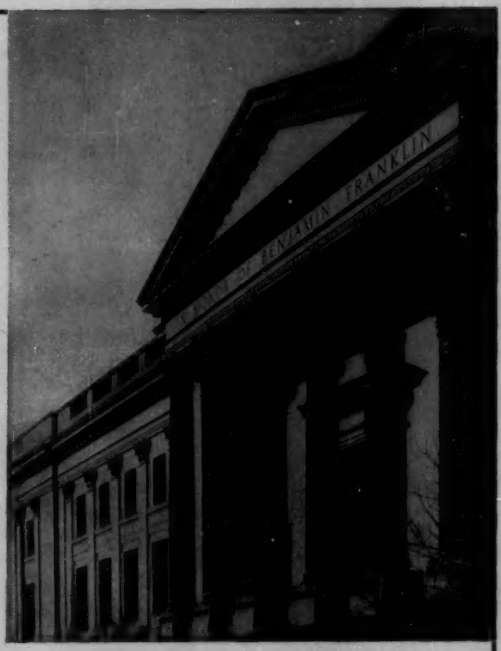

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